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Shear Resistance of Members Without Shear Reinforcement in Presence of Compressive Axial Forces in the Next Eurocode 2

Resistencia a cortante de elementos sin armadura de cortante en presencia de esfuerzos axiles de compresión en el próximo Eurocódigo 2

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ABSTRACT

The second generation of Eurocode 2 incorporates formulations based on physical models which are general enough for the assessment of existing structures but can be simplified for the design of new structures, in order to improve the ease-of-use. One of the areas where these improvements are addressed is the shear verification of members without shear reinforcement, such as solid slabs, walls, cut-and-cover tunnels, precast ribs or hollow core slabs, which in some cases are prestressed or subjected to external axial loading.

In current Eurocode 2, the shear verification of these structures is based on an empirical formulation proposed by Zsutty in 1968. The final draft of the new version of Eurocode 2 has adopted the Critical Shear Crack Theory (CSCT) as the theoretical basis for the formulation of the shear resistance, which allows a better understanding of structural behaviour in many different conditions, not only for the design of new structures, but also for the assessment of existing structures. This formulation accounts for some aspects that are not well considered in current Eurocode 2, which have been underlined as shortcomings in recent years.

The formulation for design of new structures in the final draft of the new version of Eurocode 2 (General Model) is easy to use for the verification of the shear resistance, but requires an iterative process to calculate the shear capacity of sections in the presence of axial forces. For this reason, the final draft of the new version of Eurocode 2 also provides an alternative non-iterative formulation (Linear Approach) to calculate the shear capacity in presence of compressive axial forces, based on the linearisation of the CSCT shear failure criterion and formulated with the same additive structure as in the current Eurocode 2, useful for the most common cases.

This paper presents the General Model formulation provided in the next Eurocode 2 for the shear verification of axially loaded members without shear reinforcement, as well as the alternative formulation (linear approach). In addition, the agreement of both formulations with experimental results from an available shear test database on prestressed concrete beams is shown and the consistency of the safety treatment between the two formulations is also discussed.

KEYWORDS: Shear, shear resistance, concrete structures, prestressing, axial force.

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RESUMEN

La segunda generación del Eurocódigo 2 incorpora formulaciones basadas en modelos físicos suficientemente generales para la evaluación de estructuras existentes, pero que pueden simplificarse para el diseño de nuevas estructuras, con el fin de mejorar la facilidad de uso. Uno de los ámbitos en los que se abordan estas mejoras es la verificación a cortante de elementos sin armadura de cortante, como losas macizas, muros, marcos, viguetas prefabricadas o losas alveolares, que en algunos casos están pretensados o sometidos a cargas axiales externas.

En el actual Eurocódigo 2, la verificación a cortante de estas estructuras se basa en una formulación empírica propuesta por Zsutty en 1968. El borrador final de la nueva versión del Eurocódigo 2 ha adoptado la denominada Critical Shear Crack Theory (CSCT) como

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base teórica para la formulación de la resistencia a cortante, lo que permite una mejor comprensión del comportamiento estructural en muchas condiciones diferentes, no sólo para el diseño de nuevas estructuras, sino también para la evaluación de las estructuras existentes. Esta formulación tiene en cuenta algunos aspectos que no están bien considerados en el actual Eurocódigo 2, los cuales han sido sub-rayados como deficiencias en los últimos años.

La formulación para el diseño de nuevas estructuras en el borrador final de la nueva versión del Eurocódigo 2 (Modelo General) es fácil de usar para la verificación de la resistencia a cortante, pero requiere un proceso iterativo para calcular la capacidad resistente a cortante de secciones en presencia de esfuerzos axiles. Por este motivo, el borrador final de la nueva versión del Eurocódigo 2 también proporciona una formulación alternativa que no requiere iteración (Aproximación Lineal) para calcular la capacidad resistente a cortante en presencia de esfuerzos axiles de compresión, basada en la linealización del criterio de fallo por cortante del CSCT y formulada con la misma estructura aditiva que en el actual Eurocódigo 2, útil para los casos habituales.

En este trabajo se presenta la formulación del Modelo General previsto en el próximo Eurocódigo 2 para la verificación a cortante de elementos sin armadura de cortante sometidos a esfuerzos axiles, así como la formulación alternativa (Aproximación lineal). Además, se muestra la concordancia de ambas formulaciones con los resultados experimentales de una base de datos disponible de ensayos de cortante en vigas de hormigón pretensado y la consistencia del tratamiento de seguridad entre ambas formulaciones.

PALABRAS CLAVE: Cortante, resistencia a cortante, estructuras de hormigón, pretensado, esfuerzo axial.

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1. INTRODUCTION

Many common structures, such as solid slabs, walls, cut-andcover tunnels, precast ribs, hollow core slabs, which are subjected to shear forces, are designed without shear reinforcement. These elements can be subjected to axial forces either due to prestressing or external loads.

In current Eurocode 2 [1], the shear resistance verification of members without shear reinforcement is based on an empirical formulation proposed by Zsutty in 1968 [2] from test on non-axially loaded reinforced concrete beams, but including a coefficient k to account for the size effect and an additive term to account for the axial force effect, and setting a specific value for the shear slenderness a/d. The influence of axial force on the shear resistance is based on the proposal of Hedman and Losberg [3], according to which a prestressed concrete beam has the same shear resistance as a reinforced concrete beam but adding the shear force acting when the decompression moment is reached. On the other hand, both the original Zsutty's formulation and the Hedman and Losberg proposal depend on the shear slenderness a/d. However, this influence has been removed in the current Eurocode 2 [1] by assuming a fixed value for a/d.

The availability of experimental databases with a large number of tests [4] on beams without stirrups and the research work carried out by various authors have shed light on some of the shortcomings of this formulation, which have been highlighted by Muttoni *et al.* [5] and Herbrand and Hegger[6]:

- The *k*-factor does not adequately account for the influence of the size effect. A discussion on the factor to consider the size effect in different standards can be found in [7], where it concludes that the factor proposed by ACI-446 is the one that best fits the experimental results. Compared to this factor, the *k*-factor in current Eurocode 2 [1] gives unsafe values for large values of the effective depth;
- The additive term included to consider the influence of the axial force gives too conservative or even negative values of the shear resistance in case of tensile force. This behaviour has been also investigated and pointed out by Adam and Hegger [8];

- The influence of eccentricity is not explicitly accounted for, which can lead to unsafe results for members with normal force instead of prestressing or less eccentricity (for example in members with additional external normal force or prestressing [6]);
- The influence of aggregate size on shear resistance is not considered. However, aggregate interlock, initially described by Fenwick and Paulay [9], Taylor [10] and Paulay and Loeber [11] could be the main shear transfer action in elements without shear reinforcement [12], [13] and [14]. This mechanism depends on the roughness of the shear crack and, consequently, on the aggregate size;
- The current version of Eurocode 2 [1] does not take advantage of the increase of shear resistance in members with small values of shear slenderness.

Different models have been proposed for the calculation of shear resistance in elements without shear reinforcement in the last decades. Among others: tooth models, such as those proposed by Kani [15], Hamadi and Regan [16], Reineck [17] and Yang [18]; models based on the compressed chord resistance, such as those proposed by Zararis and Papadakis [19], Hegger and Görtz [20], Park *et al.* [21] and Marí et al. [22], [23] and [24]; model based on the Critical Shear Crack Theory (CSCT) proposed by Muttoni *et al.* [25], [26], [12] and [27]; models based on fracture mechanics, such as that proposed by Carmona and Ruiz [28]; models based on crack propagation by Classen [29] and Schmidt [30]; and model based on the Modified Compression Field Theory (MCFT) proposed by Vecchio and Collins [31] and [32].

The final draft of the new version of Eurocode 2 (FprEN 1992-1-1:2023) [33] has adopted the CSCT [25] and [26] as the basis for the formulation of the detailed verification of shear and punching shear resistance in members without transverse reinforcement.

The main assumption of the CSCT [25] is that the shear stress resistance in reinforced concrete elements without shear reinforcement depends on the width and roughness of the critical shear crack developed along the web and on the concrete compressive strength (Formula (1) and Figure 1). The crack width (w) is proportional to the product of a control longitudinal strain (ε) by the effective depth of the

section (*d*). The strain ε is evaluated at a depth of 0.6*d* from the outermost compressed fibre in the critical section whose location depends on the external loading distribution. The roughness is taken into account by the aggregate size d_{g} , and $d_{g0} = 16$ mm is the reference value of the aggregate size.

$$\frac{V_R}{\sqrt{f_c} b d} = \frac{1}{3} \frac{1}{1+120 \frac{\varepsilon d}{d_c + d_c}} \tag{1}$$



Figure 1.- CSCT failure criterion and comparison to tests (adapted from [34]).

This formulation accounts for the aforementioned main parameters. The size effect and the aggregate size are directly considered and the influence of the shear slenderness and the axial force through the longitudinal strain due to the bending and axial forces concomitant with the acting shear.

This hyperbolic failure criterion has been considered in Annex I of the FprEN 1992-1-1:2023 [33] for the assessment of existing structures. However, to simplify the verification of the shear resistance of new structures, a closed-form equation has been proposed, which allows the verification of the shear resistance in a straightforward manner. Such closedform equation requires nevertheless an iterative process to calculate the shear capacity of the section in presence of axial forces. Alternatively, the FprEN 1992-1-1:2023 [33] provides a non-iterative method for the calculation of the shear capacity in presence of compressive axial forces, formulated with the same additive structure as in the current Eurocode 2 [1], useful for the most common cases.

This paper presents how the CSCT has been drafted in the next generation of Eurocode 2 to provide a General Model for the shear verification of members not requiring shear reinforcement in the presence of axial forces, as well as the alternative formulation (Linear Approach) based on the linearisation of the shear failure criterion used for the General Model. The paper also shows that both methods (General Model and Linear Approach) have similar agreement with experimental results from an available shear test database on prestressed concrete beams. On the other hand, a discussion shows the consistency on the safety treatment between both formulations. Finally, an example has been included to show the use of the general model and the linear approach in a practical case.

2. GENERAL MODEL

2.1. formulation

The formulation to calculate the shear resistance of members without shear reinforcement is based on the Critical Shear Crack Theory, using the longitudinal tensile reinforcement strain (ϵ_v) instead of the reference strain at the control depth (ϵ) [35]. According to this, the shear resistance can be expressed as the following hyperbolic law:

$$V_{R,c} = \frac{0.3}{1+48 \varepsilon_{\nu} \frac{d}{d_{d\nu}}} \sqrt{f_c} b_{\nu} d$$
(2)

where:

- ϵ_{ν} strain of the longitudinal tensile reinforcement. In case of prestressing, strain increase in the prestressing steel.
- b_w width of the cross-section
- *d* effective depth of the cross-section
- f_c compressive strength of concrete
- $\begin{array}{l} d_{dg} & = 16 \text{ mm} + D_{\text{lower}} \le 40 \text{ mm for concrete with } f_{\text{ck}} \le 60 \text{ MPa} \\ & = 16 \text{ mm} + D_{\text{lower}} \left(\frac{60}{f_{\text{ck}}} \right)^2 \le 40 \text{ mm for concrete with} \\ & f_{\text{ck}} > 60 \text{ MPa} \end{array}$

According to FprEN 1992-1-1:2022 [33], D_{lower} is the smallest value of the upper sieve size D in an aggregate for the coarsest fraction of aggregates in the concrete permitted by the specification of concrete according to EN206 [36].

The reinforcement strain $\varepsilon_{\rm V}$ can be calculated by a sectional analysis for the bending moment M_E and axial force N_E acting at the control section. To this aim, a non-linear sectional analysis can be performed.

However, in order to provide an easy-to-use formulation for design of new structures, FprEN 1992-1-1:2023 [33] introduces some simplifications:

• The hyperbolic shear failure criterion (2) is replaced by the following power-law [12],

$$V_{R,c} = k \left(\frac{f_c d_g}{\varepsilon_v d}\right)^{1/2} b_w d$$
with $k = 0.015 \left(\frac{a_{cs}}{d}\right)^{1/4}$
(3)

where a_{cs} is the effective shear span with respect to the control section.

• The reinforcement strain $\epsilon_{\rm V}$ can be calculated by a simplified flexural analysis at the control section assuming a linear elastic behaviour of the tension reinforcement (Figure 2)

$$\varepsilon_{\nu} = \frac{|M_E| + N_E e_c}{E_s A_{sl} z} = \left(1 + \frac{N_E e_c}{|M_E|}\right) \frac{|M_E|}{E_s A_{sl} z} = k_{\nu p} \frac{|M_E|}{E_s A_{sl} z}$$
(4)

Introducing the definition of the effective shear span at control section as $a_{cs}=|M_E/V_E|>=d$, the reinforcement strain is a linear function of the acting shear force.

$$\varepsilon_{\nu} = k_{\nu p} \; \frac{|V_E| \; a_{cs}}{E_s A_{sl} z} \tag{5}$$

where $k_{vp} = 1 + \frac{N_E e_c}{|M_E|}$ is a coefficient that allows to account for the effect of the axial force in the effective shear span $a_{cs,N} = k_{vp}a_{cs}$, which physically means that the presence of an compressive



c) Physical meaning of the effective shear span in presence of eccentric axial force

Figure 2. Simplified sectional analysis.

axial force reduces the length where the flexural reinforcement is in tension (Figure 2c). The value of k_{vp} is equal to 1 when there is no axial force applied, less than 1 for compressive axial forces and greater than 1 for tension axial forces. Since only tensile reinforcement strains are allowed in the shear failure criterion, k_{vp} must be greater than 0. This means that, for members subjected to compression forces, the eccentricity of the applied forces $-\frac{|M_E|}{N_E}$ must be greater than e_c . Considering $\frac{|M_E|}{-N_E} \ge \frac{e_c}{0.9}$, the minimum value of k_{vp} is 0.1.

On the other hand, the value of e_c can be approximated by a constant value equal to d/3. Therefore, the coefficient k_{vp} can be expressed as

$$k_{vp} = 1 + \frac{N_E}{|V_E|a_{cs}} \frac{d}{3} \ge 0.1$$
(6)

Replacing (5) in (3) and considering that at failure $V_E = V_{R,c}$, it follows

$$V_{R,c} = 0.015^{2/3} \left(\frac{E_s \rho_l f_c d_g}{k_{vp} \sqrt{a_{cs} d}} \frac{z}{d} \right)^{1/3} b_w d$$
(7)

where $\rho_l = \frac{A_{sl}}{b_w d}$

and considering E_s =200000 N/mm² and z/d = 0.9, Formula (7) can be rewritten as

$$V_{R,c} = 0.6 \left(\frac{100 \,\rho_l \, f_c \, d_{dg}}{k_{\nu p} \, a_{\nu}} \right)^{1/3} b_w \, d \tag{8}$$

where $a_v = \sqrt{\frac{a_{cs}d}{4}}$ is the mechanical shear span.

When the longitudinal tensile reinforcement is composed by both ordinary A_{si} and prestressed A_{pi} reinforcement located at

 d_{si} and d_{pi} respectively from the outermost compressed edge of the section, an equivalent reinforcement A_{sl} can be considered. This equivalent reinforcement provides a tensile force F_t equal to the sum of all tensile forces (increase in case of prestressing) of the reinforcements, located at a distance d from the outermost compressed edge of the section which provides a bending moment equal to the sum of all bending moments (increase in case of prestressing) of the reinforcements.

$$d = \frac{\sum A_{pi} d_{pi}^2 + \sum A_{si} d_{si}^2}{\sum A_{pi} d_{pi} + \sum A_{si} d_{si}}$$
(9)

$$A_{sl} = \frac{\sum A_{pi} d_{pi} + \sum A_{si} d_{si}}{d}$$
(10)

Figure 3 illustrates the power-law shear failure criterion and the load-deformation path, when no axial force is applied, for different values of a_{cs}/d . For a given value of a_{cs}/d , the point of intersection between the shear failure criterion (Formula (3)) and the load-deformation relationship (Formula (5)) depicts the shear resistance (Formula (8)) for this value of a_{cs}/d . Therefore, the thicker solid line that links these points depicts the relationship between the shear resistance and the reinforcement strain obtained by varying a_{cs}/d . As can be seen, the shear resistance decreases as a_{cs}/d increases. Since for $a_{cs}/d \ge 4$ the variation of the shear resistance is small, a constant value of the shear resistance can be assumed for $a_{cs}/d \ge 4$, leading to the following easy-to-use formulation included in the new Eurocode 2 (FprEN1992-1-1:2022) [33].

$$V_{R,c} = 0.6 \left(\frac{100 \ \rho_l \ f_c \ d_{dg}}{d}\right)^{1/3} b_w \ d \tag{11}$$

In presence of an axial force N_E and for a given value of $a_{cs,0}/d$ ($a_{cs,0}$ is the effective shear span when no axial force is applied,



Figure 3. Shear resistance for different a_{c/}d values when no axial force is applied.

see Figure 2c), as when there is no axial force, the shear resistance is given by the intersection between the load-deformation relationship and the shear failure criterion for this given value of $a_{c_{5}0}/d$. Figure 4 illustrates how this resistance is obtained for two values of $a_{c_{5}0}/d$. The load-deformation relationship can be expressed as a function of $a_{c_{5}0}/d$ (see Figure 2 and Formula (4))

$$V_E = -\frac{N_E (e_p + e_c)}{a_{cs,0}} + \frac{E_s A_{sl} z}{a_{cs,0}} \varepsilon_v$$

which is a straight line when a constant value of $z = 0.9 \cdot d$ is considered. The failure criterion for a given value of $a_{cs,0}/d$ can be obtained from (3) substituting a_{cs} by $a_{cs,0} + \frac{N_E e_p}{V_{R,c}}$ in the expression of k.

By repeating this process for different values $a_{cs,0}/d$, the thicker solid line in Figure 5 is obtained, which depicts the relationship between the shear resistance and the reinforcement strain for a constant value of the axial force by varying $a_{cs,0}/d$. As can be seen, each point of this line corresponds to a different value of a_{cs}/d . As in the case of no axial forces, FprEN1992-1-1:2022 [33] considers a constant value of the shear resistance for $a_{cs}/d \ge 4$.

In summary, the shear resistance, accounting for the effect of axial force and the effective shear span, can be expressed as follows

$$V_{R,c} = 0.6 \left(\frac{100 \ \rho_l \ f_c \ d_{dg}}{k_{vp} \ a_v} \right)^{1/3} b_w \ d \tag{12}$$

where:

$$k_{vp} = 1 + \frac{N_E}{|V_E| a_{cs}} \frac{d}{3} \ge 0.1$$
(13)

$$\frac{d}{2} \le a_v = \sqrt{\frac{a_{cs}d}{4}} \le d \tag{14}$$

The limits of $k_{vp} \ge 0.1$ and $a_{cs} \ge d$ (i.e. $a_v \ge d/2$) are in fact an upper bound on the shear strength.

2.2. Minimum shear resistance

Experimental evidences [37] have proven that the shear resistance decreases as the reinforcement strain increases, even when they are larger than the yielding strain. However, only in case of designs with plastic redistributions of internal forces in statically indeterminate structures, shear failures with rein-



Figure 4.- Shear resistance for different a_{cs0}/d values in presence of a compressive axial force.



Figure 5. Shear resistance for $N_E=0$ and $N_E<0$.

forcement strains larger than yielding strain can occur [38]. Therefore, for the sake of simplicity, the maximum reinforcement strain has been taken equal to the yielding strain. Thus, the minimum shear resistance can be expressed from Formula (3) as

$$V_{R,cmin} = 0.015 \left(\frac{a_{cs}}{d}\right)^{1/4} \left(\frac{E_s f_c d_{dg}}{f_y d}\right)^{1/2} b_w d$$
(15)

And considering E_s =200000 N/mm² and a value of $a_{cs}/d=4$, Formula (15) can be rewritten as

$$V_{R,cmin} = 10 \left(\frac{f_c \, d_{dg}}{f_y \, d} \right)^{1/2} b_w \, d \tag{16}$$

In case of prestressed members without ordinary reinforcement, f_y must be taken as the difference between the yielding stress and the prestress of the tendon after losses.

It is worth noting that the minimum shear resistance is not dependent on the applied axial force. This is because the minimum shear resistance is defined assuming that the flexural reinforcement yields at the load level that produces the shear failure, that is, the available flexural reinforcement is equal to that required for the bending and axial forces concomitant with the shear force applied at the control section.

On the other hand, it should also be noted that the minimum shear resistance given by (16) is not proportional to d, but to the square-root of d. This does not only mean that a size effect is being considered but also a combined size and strain



Figure 6. Verification procedure. (a) Section does not resist the applied shear. (b) Section resists the applied shear

effect. Thus, as the member size increases, the flexural reinforcement ratio required to reach the yield strength at the load level that produces the minimum shear resistance decreases.

2.3. Verification procedure and resistance capacity

When there is no axial force, k_{vp} =1 and Formula (12) becomes an explicit expression to obtain directly the shear resistance, while in presence of axial forces, k_{vp} depends on the acting shear force and Formula (12) is thus an expression in function of the acting shear force.

In a verification problem, Formula (12) can be used directly (refer to Figure 6). For given values of the applied forces $N_{\rm E}$, $M_{\rm E}$ y $V_{\rm E}$ at the control section, the effective shear span $a_{\rm cs}$ and the coefficient $k_{\rm vp}$ can be calculated, which define the straightline OA in Figure 6. The intersection of this line with the shear failure criterion (Formula (12)) for the calculated value of $a_{\rm cs}$ gives the shear resistance $V_{\rm R}$ (point B in Figure 6). If $V_{\rm E} > V_{\rm R,c}$, shear failure is predicted to occur for a shear force lower than the applied force (Figure 6(a)) and, conversely, $V_{\rm E} < V_{\rm R,c}$ means that section resists the applied shear force (Figure 6(b)).

In summary, the flow chart for the detailed verification of the shear resistance is as follows (Figure 7):

Figure 7. Flow chart for the verification of the shear resistance.

However, when the shear capacity of the control section is required, the applied shear force $V_{\rm E}$ must be taken equal to shear resistance $V_{\rm R}$. Therefore, Formula (12) becomes an equation of the unknown $V_{\rm R}$, which cannot be solved explicitly and thus needs an iterative process, as illustrated in Figure 8.



Figure 8. Iterative process to obtain the shear capacity in sections subjected to axial force.

3. LINEAR APPROACH

To avoid the iterative process described in the previous section, FprEN1992-1-1:2022 [33] provides a simplified formulation (Linear Approach) which is derived by linearizing the power law shear failure criterion used in the General Model for compressive axial forces.

3.1. Linearization of the power law shear failure criterion

In statically determinate structures subjected to a point or uniformly distributed load (Figure 9), the ratio $(a_{cs,0})$ between the bending moment and the shear force at the control section does not depend on the magnitude of the applied load.



Figure 9. Definition of the $a_{cs,0}$ in statically determinate structures



Figure 10.- Shear capacity for different compressive axial forces and a constant value of a_{cs0} (solid line: general model) (dashed line: linear approach): (a) Shear capacity vs reinforcement strain; (b) Shear capacity vs axial force.

When, in addition to the above load, a load (external or prestressing) producing an axial force N_E with an eccentricity e_p (refer to Figure 2) is acting, the ratio (a_{cs}) between the total bending moment and the applied shear force (effective shear span) depends on the axial and shear forces and is given by the following expression

$$a_{cs} = \left|\frac{M_E}{V_E}\right| = \left|a_{cs,0} + \frac{N_E e_p}{V_E}\right| \ge d$$
(17)

For a given value of $a_{cs,0}$ and N_E , the shear capacity $V_{R,c}$ is obtained by the iterative process defined in 2.3 (Figure 8) and the strain of the reinforcement (ε_V) can be calculated from (5). For different values of N_E the relationships $V_{R,c} - \varepsilon_V$ and $V_{R,c} - N_E$, keeping constant the value of $a_{cs,0}$, can be obtained (thicker solid lines in Figure 10(a) and Figure 10(b) respectively).

This thicker solid line can be linearized for compressive axial forces by the dashed line in Figure 10(b), which is defined by two points:

- Point A: which corresponds to the shear capacity of the section without applying axial force $(V_{Rc,0}, \varepsilon_{v0})$.
- Point B: which is given by the shear capacity corresponding to k_{vp} =0.1 ($V_{Rc,max}$) and ε_v =0.

This linear approach is given by the following expression:

$$V_{R,c} = V_{Rc,0} + \frac{V_{Rc,max} - V_{Rc,0}}{\varepsilon_{\nu 0}} (\varepsilon_{\nu 0} - \varepsilon_{\nu}) \le V_{Rc,max}$$
(18)
where

 $V_{Rc,0}$ is the shear resistance obtained from the General Model without considering axial force,

$$\begin{split} \varepsilon_{v0} & \text{ is the reinforcement tensile strain for } V_{\text{Rc},0}\text{, and} \\ V_{\text{Rc,max}} & \text{ is the upper limit of the shear resistance defined in the General Model by taking } k_{vp} = 0.1 \text{ and the corresponding value of } a_{cs}. \end{split}$$

The tensile strains ε_v and ε_{v0} of the longitudinal reinforcement can be obtained from the same simplified sectional analysis as that used in the General Model in section 2.1 (Figure 2 with $e_c=d/3$):

$$\varepsilon_{\nu} = \frac{V_E a_{c_s 0} + N_E \left(e_p \frac{d}{3}\right)}{A_{sl} E_s z}$$
(19)

$$\varepsilon_{v0} = \frac{V_{Rc,0} \ a_{cs,0}}{A_{sl} \ E_s \ z} \tag{20}$$

Substituting (19) and (20) in (18) and taking $V_E = V_{R,c}$, an explicit expression can be derived to calculate the shear resistance in presence of compressive axial forces

$$V_{R,c} = V_{Rc,0} - K_N N_E \le V_{Rc,max} \tag{21}$$

where

$$k_{N} = \left(1 - \frac{V_{R_{c}0}}{V_{R_{c}max}}\right) - \frac{e_{p} + \frac{a}{3}}{a_{c_{s}0}}$$
(22)

 $V_{Rc,max}$ can be approximated by the following formula (see annex 1)

$$V_{R_{c,max}} = 2.15 \left(\frac{a_{c_{s}0}}{d}\right)^{1/6} V_{R_{c}0} \le 2.71 \ V_{R_{c}0}$$
(23)



Figure 11. Histograms of the main parameters: a) Effective depth; b) shear span to effective depth ratio; c) eccentricity of axil force to effective depth ratio; d) compressive stress to concrete strength ratio.

And k_N (Formula (22)) can be simplified by taking the minimum value of $\frac{V_{R_c,max}}{V_{R_c,0}}$ =2.15", since $a_{cs,0} \ge d$. Therefore,

$$k_{\rm N} = 0.54 \frac{e_p + \frac{d}{3}}{a_{\rm cs,0}} \le 0.18 \tag{24}$$

The upper-bound of k_N is justified also in annex 1.

3.2. Linear Approach formulation

In summary, rounding the factor 0.54 in $k_{\rm N}$ to 0.5 and taking into account the minimum value of the shear resistance (Formula (16)), the Linear Approach formulation is expressed as

$$V_{Rc,min} \le V_{Rc} = V_{Rc,0} - k_N N_E \le V_{Rc,max}$$

$$\tag{25}$$

where

$$k_N = 0.5 \frac{e_p + \frac{d}{3}}{a_{c_30}} \le 0.18$$
(26)

$$V_{Rc,max} = 2.15 \left(\frac{a_{cs,0}}{d}\right)^{1/6} V_{Rc,0} \le 2.71 \ V_{Rc,0}$$
(27)

4.

COMPARISON WITH EXPERIMENTAL TEST RESULTS

4.1. Selected database

A comparison of the shear resistance calculated using the General Model and the Linear Approach with the experimental results of a selected database (see annex 2) has been performed in order to check the agreement between both formulations. This selected database is based on the ACI-DAfStb-Database of PC beams without stirrups (vuct-PC) [39][40], removing the tests for which no aggregate size is provided, those reported as flexural or anchorage failure and those with shear reinforcement, and adding the tests reported by De Wilder et al. [41] as well as Joergensen and Fisker [42].

The total number of tests is 183, 85 with rectangular cross-section and 98 with profiled cross-section. All tests are subjected to compressive axial forces.

Table 1 and Figure 11 present the range and the histogram of each of the main parameters.

TABLE 1. Range of the main parameters.				
Parameter	I			

Parameter	Min	Max
d [mm]	109	1025
a/d [-]	2.42	7.30
e _p /d [-]	0.13	0.51
$ \sigma_{\rm cp} /f_{\rm c}$ [-]	0.004	0.258

4.2. Statistical values

The formulations described in section 2 for the General Model (GM) (Formulae (12)-(14)) and in section 3 for the Linear Approach (LA) (Formulae (25)-(27)), taking into account the minimum value of the shear resistance given by Formula (16), as well as the current EN1992-1-1:2004 [1], have been ap-

Table 2. Statistical values for the ratio experimental to predicted shear strength ($V_{\rm test}/V_{\rm cal})$

		All tests		Re	ctangular section	ons	Profiled sections				
Num		183			85		98				
Method	EC2	GM	LA	EC2	GM	LA	EC2	GM	LA		
AVG	1.59	1.52	1.51	1.32	1.47	1.40	1.84	1.56	1.60		
SD	0.481	0.367	0.365	0.408	0.415	0.408	0.403	0.315	0.297		
CoV	0.302	0.242	0.242	0.311	0.282	0.291	0.220	0.203	0.186		
Max	3.40	2.67	2.59	2.39	2.67	2.59	3.40	2.51	2.54		
Min	0.70 0.82 0.7		0.78	0.70	0.70 0.82		1.00	0.91	1.01		
$V_{test}/V_{cal} \le 1$	17	5	4	17	3	4	0	0			

EC2: Current Eurocode 2 [1] GM: General Model

LA: Linear Approach

plied to the tests in the selected database to obtain the ratio experimental to predicted shear strength ($V_{\text{test}}/V_{\text{cal}}$). The statistical values of this ratio are the followings:

As can be seen in Table 2, the statistical values of $V_{\text{test}}/V_{\text{cal}}$ provided by the General Model (GM) and Linear Approach (LA) are almost the same for all tests and very similar when they are evaluated for rectangular sections or profiled sections separately. On the other hand, the values of CoV obtained with current Eurocode 2 [1] are higher than those provided by GM and LA formulations, both for rectangular and profiled sections and especially for all tests. Furthermore, the difference between the mean values calculated for rectangular sections and for profiled sections is much higher when using the current Eurocode 2 than when GM and LA formulations are used.

It should be noted that the tests where shear failures do not develop from a flexural crack have not been removed from the selected database. When these formulations are applied to these tests the calculated values are much less than the test results. If the 19 tests with $V_{test}/V_{cal}>2$ are removed from the 183 tests of the selected database, CoV decrease significantly from 0.242 to 0.172, which is in line with the CoV obtained for members without axial force [5].

Figure 12 shows the experimental results against those provided by current Eurocode 2 [1], GM and LA formulations. As can be seen, the scatter and the R^2 coefficient is practically the same for GM and LA formulations and this R^2 coefficient (0.88) is better than that of the current Eurocode 2 (0.76).

Figure 13 to Figure 16 show the ratio V_{test}/V_{cal} as a function of the main parameters. As can be seen, the General model and the Linear Approach give similar results and trends for all these parameters.

In addition, it is worth noting that these formulations provide improvements with respect to current Eurocode 2 related to the influence of shear span to effective depth ratio and axial force on the shear resistance in presence of compressive axial force. While the current EC2 gives V_{test}/V_{cal} values below 1 for a/d>6 and very conservative for small values of a/d, GM and LA formulations better capture the influence of this variable (Figure 14). Likewise, Figure 16 shows that the current EC2 gives many V_{test}/V_{cal} values below 1 for Σ_{cp}/f_c between 0.05 and 0.15 and is very conservative for high values of this variable. However, GM and LA captures more accurately the influence of this variable.



Figure 12. V_{test} vs. V_{cal}. a) Eurocode 2 (2004). b) General Model. c) Linear approach.



Figure 13.- Ratio V_{test}/V_{cal} vs. effective depth. a) Eurocode 2 (2004). b) General Model. c) Linear approach.



Figure 14. Ratio V_{test}/V_{cal} vs. shear span to effective depth ratio. a) Eurocode 2 (2004). b) General Model. c) Linear approach.



Figure 14. Ratio V_{test}/V_{cal} vs. shear span to effective depth ratio. a) Eurocode 2 (2004). b) General Model. c) Linear approach.



Figure 15. Ratio V_{test}/V_{cal} vs. eccentricity of axial force to effective depth ratio. a) Eurocode 2 (2004). b) General Model. c) Linear approach.



Figure 16. Ratio V_{test}/V_{cal} vs. compressive stress to concrete strength. a) Eurocode 2 (2004). b) General Model. c) Linear approach.

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5. SAFETY FORMAT FORMULATION

The design shear resistance power law criterion can be expressed from (3) using a partial safety factor γ_R

$$\frac{V_{Rd,c}}{b_w d} = \frac{1}{\gamma_R} \frac{V_{Rk,c}}{b_w d} = \frac{k}{\gamma_R} \left(\frac{f_{ck} d_{dg}}{\varepsilon_{vd} d}\right)^{1/2} \le \frac{V_{Rdc,max}}{b_w d}$$
(28)

In absence of axial forces, a partial factor, γ_{dep} can be assumed to account for the uncertainties related to the calculation of the deformation. Therefore, the design simplified load-deformation relationship can be obtained from (5) with k_{vp} =1 and applying the partial factor γ_{def}

$$\varepsilon_{vd} = \gamma_{def} \frac{|V_{Ed}| a_{cs,0}}{E_s A_{sl} z}$$
⁽²⁹⁾

The design value of the shear resistance $V_{Rd,c}$ can be obtained by substituting (29) in (28) and making $V_{Ed}=V_{Rd,c}$

$$\frac{V_{Rd,c}}{b_w d} = \frac{0.60}{\gamma_R^{2/3} \gamma_{def}^{1/3}} \left(100 \,\rho_l \, f_{ck} \, \frac{d_{dg}}{\sqrt{\frac{a_{cs,0}}{4} d}} \right)^{1/3} \tag{30}$$

The product $\gamma R^{2/3} \cdot \gamma_{def}^{1/3}$ can be considered as a single safety factor γ_{V} , which involves all the uncertainties related to the model, the material and the geometry. So, the expression (30) can be written as

$$\frac{V_{Rd,c}}{b_{w}d} = \frac{0.60}{\gamma_{V}} \left(100 \,\rho_{l} \, f_{ck} \, \frac{d_{dg}}{\sqrt{\frac{a_{cs}0}{4}d}} \right)^{1/3}$$
(31)

The partial safety factor γ_V is calibrated according to annex A of FprEN 1992-1-1:2023 [33] and the background document to Annex A [43] on the basis of the statistical values of the most sensitive random variables appearing in this Formula.

In presence of axial force, as in the case without axial force, to take into account the uncertainties related to the reinforcement strain, a partial safety factor γ_{def} could be assumed. Therefore, the design value ε_{vd} can be expressed from (5), as

$$\varepsilon_{\nu d} = \gamma_{def} \frac{|V_{Ed}| \ a_{cs} k_{\nu p}}{E_s A_{sl} z}$$
(32)

where
$$k_{vp} = 1 + \frac{N_E}{|V_E|a_{cs}} \frac{d}{3} \ge 0.1$$
 and $a_{cs} = \left|\frac{M_{Ed}}{V_{Ed}}\right| \ge d$

To calculate the design value of the shear resistance $V_{Rd,c}$ by means of Formula (28), the design value of the applied shear force V_{Ed} must be taken equal to $V_{Rd,c}$.

$$V_{d,c} = \frac{0.6}{\gamma_V} \left(\frac{100 \rho_l f_c d_{dg}}{k_{vp} a_v} \right)^{1/3} b_w d$$
(33)

Since the coefficients k_{vp} and a_v are function of V_{Edv} Formula (33) becomes a non-linear equation of $V_{Rd.c}$. Thus, unlike the case without presence of axial force, this Formula is not an explicit function of the random variables that governs this mode of failure. However, for the sake of simplicity FprEN 1992-1-1:2023 [33] assumes the same value of the partial factor γ_V as for members without axial force. This assumption is conservative since the scatter of the ratio V_{test}/V_{cal} decreases as γ_{cp}/f_c increases, as can be seen in Figure 16b.

It should be noted that the ratio $V_{Rk,c}/V_{Rd,c}$, which results by applying the General Model with $\gamma_V=1$ for $V_{Rk,c}$ and γ_V for $V_{Rd,c}$, is variable with N_{Ed} because Formula (33) is a non-linear equation.

Similarly, in the Linear Approach, the design shear failure criterion is obtained from (18) using the design values of the shear resistance

$$V_{Rd,c} = V_{Rdc,0} + \frac{V_{Rdc,max} - V_{Rdc,0}}{\varepsilon_{vd0}} \left(\varepsilon_{vd0} - \varepsilon_{vd}\right) \ge V_{Rdc,max}$$
(34)

The design simplified load-deformation relationship used in this approach is the same expression (32) as that used by the General Model

$$\varepsilon_{vd} = \gamma_{def} \frac{|V_{Ed}| a_{cs} k_{vp}}{E_s A_{sl} z} = \gamma_{def} \frac{|V_{Ed}| a_{cs,0} + N_{Ed} \left(e_p + \frac{a}{3}\right)}{E_s A_{sl} z}$$
(35)

The tensile reinforcement strain ε_{vd0} is given by

$$\varepsilon_{vd0} = \gamma_{def} \frac{|V_{Rdc,0}| a_{cs,0}}{E_s A_{sl} z}$$
(36)

The shear capacity $V_{Rd,c}$ can now be obtained by substituting (36) and (35) in (34) and making $V_{Ed}=V_{Rd,c}$

$$V_{Rd,c} = V_{Rdc,0} - \frac{V_{Rdc,max} - V_{Rdc,0}}{V_{Rdc,max}} \frac{e_p + \frac{d}{3}}{a_{cs,0}} N_{Ed} \ge V_{Rdc,max}$$
(37)

Taking into account that $\frac{V_{Rdc,max} - V_{Rdc,0}}{V_{Rdc,max}} = \frac{V_{Rc,max} - V_{Rc,0}}{V_{Rc,max}}$

and $\frac{V_{Rc,max}}{V_{Rc,0}} = 2.154 \left(\frac{a_{cs,0}}{d}\right)^{1/6} \le 2.71$ (see annex 1), Formula (37) can be written as

$$V_{Rd,c} = V_{Rdc,0} - k_N N_{Ed} \le 2.15 \left(\frac{a_{cs,0}}{d}\right)^{1/6} V_{Rdc,0} \le 2.71 V_{Rdc,0}$$
(38)

where k_N is given by Formula (26). Therefore, the coefficient $k_N = 0.54 \frac{e_p + \frac{d}{3}}{a_{cs,0}}$ that multiplies N_{Ed}

does not include any partial safety factor.

6.

EXAMPLE OF APPLICATION

Figure 17 presents a simply supported prestressed beam subjected to two point-loads used as an example for the detailed shear verification by the General Model (see Figure 6) and for the calculation of the shear capacity by both the General Model (see Figure 8) and the Linear Approach.



Figure 17. Example of application.

Geometrical data (mm)

L	а	h	b	e _p	c _b	c _t	A_{sb}	A _{st}	A _p
10000	4000	700	250	150	60	60	942	628	1050

Material data:

- Concrete: $f_{ck} = 60$ MPa. $D_{lower} = 16$ mm
- Reinforcement: f_{vk} =500 MPa
- Prestressing steel: $f_{py} = 1560$ MPa

Design value of the prestressing force: $P_d = 1155$ kN. External load: $F_{Ed} = 200$ kN

6.1. Preliminary calculations:

Effective depth of the ordinary reinforcement:

 $d_s = h - c_b = 700 - 60 = 640 \text{ mm}$

Effective depth of the prestressed reinforcement:

 $d_p = \frac{h}{2} + e_p = 350 + 200 = 550 \text{ mm}$

Effective depth:

$$d = \frac{d_s^2 A_s + d_p^2 A_p}{d_s A_s + d_p A_p} = \frac{640^2 \ 942 + 500^2 \ 1050}{640 \ 942 + 500 \ 1050} = 575 \ mm$$

Size parameter:

$$d_{dg} = 16 + D_{lower} = 16 + 16 = 32 \text{ mm} \le 40 \text{ mm}$$

Reinforcement ratio:

$$\rho_l = \frac{d_s A_s + d_p A_p}{d_w d^2} = \frac{640 \ 942 + 500 \ 1050}{250 \ 575^2} = 0.01365$$

Minimum shear stress resistance:

$$\tau_{Rdc,min} = \frac{V_{Rdc,min}}{b_w z} = \frac{11}{\gamma_v} \sqrt{\frac{f_{ck}}{f_{yd}}} \frac{d_{dg}}{d} = \frac{11}{1.4} \sqrt{\frac{60}{\frac{500}{1.15}}} \frac{32}{575} = 0.689 \text{ MPa}$$

The control section is located at a distance from the support axis.

In this control section, the design forces are:

 $N_{Ed} = -P_d = -1100 \text{ kN}$

 $M_{Ed} = F_{Ed} x - P_d e_p = 200 \ 3.45 - 1100 \ 0.15 = 520 \ \text{kN m}$

$$\tau_{Ed} = \frac{V_{Ed}}{b_w z} = \frac{200000}{250 \ (0.9 \ 575)} = 1.54 \text{ MPa}$$

Since is greater than, detailed verification of the shear resistance cannot be omitted.

6.2. Verification procedure using the General Model (see Figure 6)

The design value of the shear stress resistance can be obtained by dividing Formula (33) by $b_w z$ and taking d/z=1.1.

$$\tau_{Rd,c} = \frac{0.66}{\gamma_{\nu}} \left(100 \,\rho_l \, f_{ck} \, \frac{d_{dg}}{k_{\nu p} \, a_{\nu}} \right)^{1/3} = \tau_{Rdc,min}$$

where:

$$k_{vp} = 1 + \frac{N_{Ed}}{|V_{Ed}|} \frac{d}{3 a_{cs}} = 1 + \frac{-1000}{200 \ 2600} \frac{575}{3} = 0.595 \ge 0.1$$

$$a_v = \sqrt{\frac{a_{cs,0}}{4}d} = \sqrt{\frac{2600}{4}575} = 611 \,\mathrm{mm} \ge d = 575 \,\mathrm{mm}$$

with

$$a_{cs} = \left| \frac{M_{Ed}}{V_{Ed}} \right| = \frac{575}{200} \ 1000 = 2600 \,\mathrm{mm} \ge d = 575 \,\mathrm{mm}$$

By substituting these values:

$$\tau_{Rd,c} = \frac{0.66}{\gamma_{\nu}} \left(100 \ 0.01365 \ 60 \ \frac{32}{0.595 \ 575} \right)^{1/3} = 0.929 \text{MPa} \ge \tau_{Rdc,min} = 0.689 \text{MPa}$$

Since τ_{Ed} = 1.54 MPa is greater than $\tau_{Rd,c}$ = 0.929 MPa, shear reinforcement should be provided in this control section.

6.3. Shear capacity using the General Model (see Figure 8)

The calculations performed in this section are not needed when it deals with a verification problem, because they do not change the result of this verification related to the calculation performed in the section 6.2. They are only needed when the value of the shear capacity of the section is required.

To obtain the shear capacity in the control section, as indicated in section 2.3, the design shear force V_{Ed} must be equal to the design value of the shear resistance $V_{\text{Rd.c}}$, which requires an iterative process applying the expressions given in section 6.2, since k_{vp} and a_{cs} depend on the design shear force. The result of this iterative process gives the following results:

x	$V_{\scriptscriptstyle Ed}$	\mathbf{M}_{Ed}	a _{cs}	a _v	k _{vp}	$\boldsymbol{\tau}_{_{Rd,c}}$	V _{Rd,c}
m	kN	kN∙m	m	m	[-]	MPa	kN
3.425	143.311	325.86	2.274	0.572	0.353188	1.108	143.311

As can be seen, the shear capacity is 1.108 MPa.

6.4. Shear capacity using the Linear Approach

The design value of the shear stress resistance can be obtained by dividing Formula (38) by $b_w \cdot z$ and taking d/z=1.1:

$$\tau_{Rdc,min} \leq \tau_{Rd,c} = \tau_{Rdc,0} - k_1 \,\sigma_{cp} \leq \tau_{Rdc,max}$$

where:

$$\tau_{Rdc,0} = \frac{0.66}{\gamma_{\nu}} \left(100 \,\rho_l \, f_{ck} \, \frac{d_{dg}}{a_{\nu,0}} \right)^{1/3} = 0.782 \, \text{MPa}$$

$$k_1 = \frac{0.5}{a_{c_{5,0}}} \left(e_p + \frac{d}{3} \right) \frac{A_c}{b_w z} = 0.067 \le 0.18 \frac{A_c}{b_w z} = 0.185$$

$$\sigma_{cp} \le \frac{N_{Ed}}{A_c} = -6.286 \text{ MPa}$$

$$\tau_{Rdc,max} = 2.15 \ \tau_{Rdc,0} \left(\frac{a_{cs,0}}{d}\right)^{1/6} = 2.264 \text{ MPa} \le 2.71 \ \tau_{Rdc,0} = 2.119 \text{ MPa}$$

 $a_{cs,0} = a - d = 3425 \text{ mm} \ge d = 575 \text{ mm}$

$$a_{v,0} = \sqrt{\frac{a_{cs,0}}{4}d} = 702 \text{ mm} \neq d = 575 \text{ mm}$$

By substituting these values:

$$\tau_{Rdc,min} = 0.689 \text{MPa} \le \tau_{Rdc} = 0.782 + 0.067 \cdot 6.286 = 1.203 \text{MPa} \le \tau_{Rdc,max} = 2.119 \text{MPa}$$

The shear capacity is 1.20 MPa, which is lower than τ_{Ed} =1.54 MPa. Thus, shear reinforcement should be provided in this control section.

7. CONCLUSIONS

- 1. The final draft of the new version of Eurocode 2 provides a General Model formulation to calculate the shear resistance of members without shear reinforcement in the presence of axial forces (prestressing or external load) based on the Critical Shear Crack Theory, as a theoretical extension of the formulation of shear resistance without axial force, by including a single coefficient k_{vp} .
- 2. Another feature of the final draft of the new version of Eurocode 2 is the explicit incorporation of the influence of the shear slenderness in the formulation of the shear resistance for members without shear reinforcement.
- 3. In addition, a new partial safety factor γ_v has been introduced. This partial factor account for both the uncertainties of the variables involved in the shear resistance and the model uncertainties, allowing it to be adjusted to appropriate values by means of Annex A provisions of the final draft of the new version of Eurocode 2.
- 4. The General Model formulation is easy to use in practice for verification problems in presence of axial force both for tension and compression, although it requires an iterative process when the shear capacity is required.
- 5. Alternatively, the final draft of the new version of Eurocode 2 provides a Linear Approach formulation to calculate the shear resistance in presence of compressive axial forces that is derived by linearising the shear failure criterion based on the Critical Shear Crack Theory and is therefore consistent with the General Model.
- 6. This Linear Approach allows to calculate the shear capacity without iteration for the most practical common cases.
- 7. The agreement of the shear resistance predicted by both formulations (General Model and Linear Approach) to the experimental results are similar and have a lower dispersion than that provided by the current Eurocode 2.
- 8. The shear resistance formulation provided by the Linear Approach has a similar format to that of the current Eurocode 2, although it introduces the main variables on which the axial effects on the shear resistance depends: the shear slenderness, the eccentricity of the axial force and the shape of the section.

Notation

- A_c Cross-sectional area of concrete
- A_{pi} Cross-sectional area of longitudinal prestressed reinforcement *i* located in the tensile zone due to bending at a distance d_{pi} from the outermost compressed fibre of the cross-section.
- A_{sl} Equivalent cross-sectional area of the reinforcement in the tensile zone due to bending.
- $D_{\rm lower}$ Smallest value of the upper sieve size in an aggregate for the coarsest fraction of aggregates in the concrete permitted by the specification of concrete
- *E*_s Modulus of elasticity of the flexural reinforcement
- $M_{\rm E}$ Acting bending moment
- $M_{\rm E,0}$ Acting bending moment without considering the effect of prestressing or external load that produces axial forces
- N_E Applied axial force
- $N_{\rm Ed}$ Design value of the applied axial force
- V_E Applied shear force
- $V_{\rm Ed}$ Design value of the applied shear force
- $V_{\rm R,c}$ Shear resistance of members without shear reinforcement
- $V_{\text{Rc},0}$ Shear resistance of members without shear reinforcement without considering the effect of prestressing or external load that produces axial force
- $V_{\rm Rc,max}$ Maximum shear resistance of members without shear reinforcement
- $V_{\rm Re,min}$ Minimum shear resistance of members without shear reinforcement
- $V_{\rm Rd,c}$ Design value of the shear resistance of members without shear reinforcement
- $V_{\rm Rdc,0}$ Design value of the shear resistance of members without shear reinforcement without considering the effect of prestressing or external load that produces axial forces
- $V_{\rm Rdc,max}$ Design value of the maximum shear resistance of members without shear reinforcement
- $V_{\rm Rk,c}$ Characteristic value of the shear resistance of members without shear reinforcement
- a_{cs} Effective shear span with respect to the control section
- $a_{\rm es,0}$ Effective shear span with respect to the control section without considering the effect of prestressing or external load that produces axial forces
- *a*_v Mechanical shear span
- $b_{\rm w}$ Minimum width of the cross-section between tension and compression chords
- d Effective depth of a cross-section
- d_{pi} Distance from outermost compressed fibre of the cross-section to the prestressed reinforcement *i*

- $d_{\rm si}$ Distance from outermost compressed fibre of the cross-section to the ordinary reinforcement *i*
- e_c Distance from centroid of the cross section to the resultant of the normal compressive stresses.
- e_p Eccentricity of the axial forces related to the centroid of the cross-section, positive when the eccentricity is on the side of the flexural reinforcement in tension
- $f_{\rm c}$ Cylinder compressive strength of concrete
- $f_{\rm ck}$ Characteristic cylinder compressive strength of concrete
- k_1 Coefficient to take into account the influence of axial forces on the shear stress resistance in the Linear approach
- $k_{_{\rm N}}$ Coefficient to take into account the influence of axial forces on the shear resistance in the Linear approach
- $k_{\rm vp}$ Coefficient to take into account the influence of axial forces on the shear resistance in the General Model
- z Inner lever arm of internal forces
- γ_{def} Partial safety factor which covers the uncertainties related to the calculation of the strain in the longitudinal tensile reinforcement
- γ_R Partial safety factor which covers the uncertainties related to the shear failure criterion
- γ_{v} Partial safety factor for shear resistance without shear reinforcement
- ϵ_v Strain in the longitudinal tensile reinforcement
- ϵ_{vd} Design value of the strain in the longitudinal tensile reinforcement
- $\epsilon_{v0} \qquad \qquad \text{Strain in the longitudinal tensile reinforcement without} \\ \text{considering the effect of prestressing or external load} \\ \text{that produces axial forces} \end{cases}$
- ϵ_{vd0} Design value of the strain in the longitudinal tensile reinforcement without considering the effect of prestressing or external load that produces axial forces
- $\sigma_{_{cp}}$ Compressive stress in the concrete from axial load or prestressing

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- (*) This document is available through the National members at CEN TC250/SC2 Eurocode 2

ANNEX **1** Derivation of $V_{R_{c,max}}$ and the upper bound of coefficient k_N in the Linear Approach

Expression for V_{Rcmax}

The maximum shear resistance can be expressed by Formula (12) for the shear resistance of the general model by taking the minimum values for k_{vp} and a_{cs} (k_{vp} =0.1 and a_{cs} =d)

$$V_{R_{c,max}} = 0.6 \left[100 \,\rho_l \, f_c \, \frac{d_{dg}}{0.1 \, \frac{d}{2}} \right]^{1/3} b_w \, d \tag{39}$$

On the other hand, the shear resistance obtained from the General Model without considering axial force is given by Formula (12) by taking $k_{\rm w}$ =1

$$V_{R_{c}0} = 0.6 \left(100 \,\rho_l \, f_{ck} \, \frac{d_{dg}}{\sqrt{\frac{a_{cs0} \, d}{4}}} \right)^{1/3} \!\!\! b_w \, d \tag{40}$$

Dividing (39) by (40) it follows

$$\frac{V_{R_c,max}}{V_{R_c0}} = 2.15 \left(\frac{a_{cs0}}{d}\right)^{1/6} \le 2.71$$
(41)

which has an upper limit of 2.71 because $\frac{a_{cs,0}}{d} \leq 4$

Upper bound of $k_{_N}$

The coefficient

$$k_N = 0.54 \frac{e_p + \frac{d}{3}}{a_{cs0}}$$

is upper-bounded to 0.18 because the condition $a_{cs} \ge d$ entails that

$$\frac{e_p + \frac{d}{3}}{a_{cs,0}} \le \frac{1}{3}$$

This condition can be easily understood by means of Figure 18.

As can be seen in the graph on the left of Figure 18, for a given value of $a_{cs,0}$, for different compressive axial forces, the corresponding shear resistances are obtained by intersection of the linearized failure criterion (thinner solid line) with the load-deformation relationships from Formula (19) (thicker solid lines), which can be rewritten in function of a_{cs} as

$$\varepsilon_{\nu} = \frac{V_E \ a_{cs} + N_E \frac{d}{3}}{E_s \ A_{sl} \ z}$$
(42)

When the compressive axial force increases, a_{cs} decreases and it can become equal to the effective depth for a certain value of the compressive axial force N^* . For $N_E < N_E^*$, the load-deformation relationship changes and becomes defined by the condition $a_{cs} = d$, which is expressed by

$$\varepsilon_{\nu} = \frac{V_E d + N_E \frac{d}{3}}{E_s A_{sl} z}$$
(43)

and depicted with the thicker dashed lines in Figure 18, which have different slope than the thicker solid lines.

The graph on the right in Figure 18 shows the relationship between the shear resistance and the axial force, which is defined by two segments: A-B (from 0 to $-N_E^*$) given by Formula (42) and B-C (from $-N_E^*$ to $-3 V_{R_C,max}$) given by Formula (43). This bilinear law is simplified by the thicker solid line (A-C) which slope is

$$k_{N,max} = \frac{V_{Rc,max} - V_{Rc,0}}{3 V_{Rc,max}}$$
(44)

The minimum value of $k_{N,max}$ is equal to 0.18 since the minimum value of $\frac{V_{Rc,max}}{V_{Rdc0}}$ is 2.154, according to Formula (41) for $a_{cs0}=d$.



Figure 18.- Upper bound of coefficient k_N .

ANNEX **2** Selected database

	Concrete cross section			Reinfo	Reinforcement			Pres	Cor	ıcrete	LOAD				
Notation	Туре	A	b _w	d	A	d _p	A _p	f _{py}	Р	e _p	$\sigma_{\rm p}$	f	D _{lower}	a	V _{test}
		mm ²	mm	mm	mm ²	mm	mm ²	MPa	kN	mm	MPa	MPa	mm	mm	kN
Arthur_1965_002_A2	Р	21935	51	0	0	201	77	1459	-91.2	47.4	924	31.44	9.5	914	25.4
Arthur_1965_027_B1	P	25806	51	0	0	272	142	1334	-150.3	64.6	824	45.74	9.5	914	45.9
Arthur_1965_028_B2	P	25806	51	0	0	272	142	1334	-145.9	64.6	800	42.30	9.5	914	49.0
Arthur_1965_029_B3	P	25806	51	0	0	272	142	1334	-143.2	64.6	785	46.88	9.5	686	66.4
Arthur_1965_031_B5	P	25806	51	0	0	272	142	1334	-160.1	64.6	878	51.45	9.5	686	66.4
Arthur_1965_034_B8	P	25806	51	0	0	272	142	1334	-145.9	64.6	800	51.45	9.5	686	64.2
Arthur_1965_035_B9	P	25806	51	0	0	272	142	1334	-143.2	64.6	785	42.30	9.5	914	38.8
Arthur_1965_054_E1	P	30968	76	0	0	272	142	1334	-142.3	64.6	780	45.16	9.5	762	58.1
Arthur_1965_055_E2	P	30968	76	0	0	272	142	1334	-145.9	64.6	800	51.45	9.5	762	67.4
Elzanaty_1985_001_CW1	P	54193	51	432	214	369	568	1749	-606.7	140.3	1069	76.55	12.7	1071	137.7
-	P	54193	51	432	214	369	568	1749	-596.5	140.3	1003	76.55	12.7	1847	117.4
Elzanaty_1985_002_CW3	P	54193	51	432	214	369	568	1749	-603.1	140.3	1051	76.55	12.7	1385	117.4
Elzanaty_1985_003_CW2	P	-					+				-	+			-
Elzanaty_1985_005_CW5	P	54193	51	432	1164	369	568	1749	-605.8	140.3	1067	77.93	12.7	1385	124.1
Elzanaty_1985_006_CW7		54193	51	432	214	369	395	1799	-443.9	140.3	1124	77.59	12.7	1385	105.9
Elzanaty_1985_007_CW6	P	54193	51	432	214	369	568	1749	-455.0	140.3	801	77.93	12.7	1385	112.1
Elzanaty_1985_008_CW9	P	54193	51	432	214	369	568	1749	-447.0	140.3	787	61.03	12.7	1385	101.0
Elzanaty_1985_009_CW8	P	54193	51	432	214	369	568	1749	-451.5	140.3	795	41.38	12.7	1385	89.8
Elzanaty_1985_010_CI1	P	58710	76	330	214	242	568	1749	-604.9	102.1	1066	76.55	12.7	1845	77.8
Elzanaty_1985_013_CI4	Р	58710	76	0	0	242	568	1749	-610.3	102.1	1075	78.62	12.7	1372	108.5
Elzanaty_1985_014_CI5	Р	58710	76	330	1164	242	568	1749	-604.5	102.1	1065	77.93	12.7	1372	119.7
Elzanaty_1985_015_CI7	Р	58710	76	330	214	242	395	1799	-443.0	102.1	1122	77.59	12.7	1372	81.4
Elzanaty_1985_016_CI6	Р	58710	76	330	214	242	568	1749	-455.0	102.1	801	77.93	12.7	1372	89.0
Elzanaty_1985_017_CI9	Р	58710	76	330	214	242	568	1749	-446.6	102.1	787	61.03	12.7	1372	87.2
Evans_1963_002_S2	R	30968	102	0	0	251	514	621	-58.3	99.1	113	38.83	6.4	610	106.1
Evans_1963_019_S19	Р	23226	51	0	0	257	506	621	-91.6	104.1	181	30.00	6.4	940	48.1
Evans_1963_025_S25	Р	23613	53	0	0	257	388	621	-92.5	104.1	238	36.90	6.4	940	53.5
Evans_1963_027_S27	Р	23226	51	0	0	257	508	621	-86.3	104.1	170	35.38	6.4	711	66.7
Evans_1963_029_S29	Р	23226	51	0	0	257	388	621	-81.8	104.1	211	28.55	6.4	711	64.9
Evans_1963_043_S43	Р	11177	56	0	0	122	506	621	-52.0	46.0	103	48.34	6.4	508	29.0
Evans_1963_046_S46	R	12000	79	0	0	109	77	1241	-52.5	33.0	680	33.03	6.4	508	17.7
Kar_1968_001_A1	R	32258	127	0	0	178	101	1386	-80.1	50.8	790	35.93	19.1	889	27.1
Kar_1968_003_A4	R	32258	127	0	0	178	194	1476	-82.7	50.8	427	28.83	19.1	622	55.6
Kar_1968_004_A5	R	32258	127	0	0	178	194	1476	-104.8	50.8	541	34.48	19.1	533	69.3
Kar_1968_005_A6	R	32258	127	0	0	178	155	1476	-48.9	50.8	316	28.03	19.1	711	38.9
Kar_1968_006_A7	R	32258	127	0	0	178	194	1476	-82.6	50.8	427	30.21	19.1	686	45.8
Kar_1968_007_A8	R	32258	127	0	0	178	194	1476	-125.9	50.8	650	34.14	19.1	737	43.8
Kar_1968_008_A9	R	32258	127	0	0	178	194	1476	-145.8	50.8	753	33.79	19.1	686	54.6
Kar_1968_009_A10	R	32258	127	0	0	178	194	1476	-145.8	50.8	753	31.79	19.1	889	40.8
Kar_1968_010_A12	R	32258	127	0	0	178	155	1476	-102.7	50.8	664	34.90	19.1	711	44.3
Kar_1968_011_B3	R	20645	102	0	0	152	155	1476	-48.9	50.8	316	29.17	19.1	533	28.8
Kar_1968_012_B4	R	20645	102	0	0	152	155	1476	-48.3	50.8	312	32.00	19.1	610	29.3
Kar_1968_013_B5	R	20645	102	0	0	152	155	1476	-41.2	50.8	266	28.03	19.1	686	25.8
Kar_1968_014_B6	R	20645	102	0	0	152	194	1476	-59.6	50.8	308	30.21	19.1	711	27.3
Kar_1968_015_B7	R	20645	102	0	0	152	155	1476	-65.5	50.8	423	33.17	19.1	533	38.6
Kar_1968_016_B9	R	20645	102	0	0	152	155	1476	-65.5	50.8	423	33.31	19.1	762	26.3
Kar_1968_017_B10	R	20645	102	0	0	152	155	1476	-82.7	50.8	534	35.45	19.1	762	33.7
Kar_1968_018_I-10	P	35806	76	0	0	229	194	1476	-104.5	76.2	540	35.34	19.1	756	69.3

	Concr	ete cross s	section	Reinfo	rcement			Pres	Cor	ocrete	LOAD				
Notation	Туре	A _c	b _w	ds	A _s	d _p	A _p	f _{py}	Р	e _p	$\sigma_{\rm p}$	f _c	D _{lower}	a	V _{test}
		mm ²	mm	mm	mm ²	mm	mm ²	MPa	kN	mm	MPa	MPa	mm	mm	kN
Kar_1968_019_I-11	Р	35806	76	0	0	229	194	1476	-126.2	76.2	652	38.62	19.1	756	71.7
Kar_1968_020_I-14	Р	35806	76	0	0	229	194	1476	-145.5	76.2	752	34.48	19.1	756	69.8
Kar_1968_021_I-15	Р	35806	76	0	0	216	194	1476	-83.2	63.5	430	30.41	19.1	540	66.5
Kar_1968_024_I-19	Р	35806	76	0	0	229	194	1476	-146.3	76.2	756	33.79	19.1	756	64.4
Kar_1968_025_I-20	Р	35806	76	0	0	216	194	1476	-106.8	63.5	552	35.17	19.1	610	62.0
Kar_1968_026_I-21	Р	35806	76	0	0	229	194	1476	-127.7	76.2	660	35.17	19.1	864	49.7
Kar_1968_029_D3	Р	28064	51	0	0	216	194	1476	-123.6	63.5	638	30.76	19.1	540	74.6
Kar_1968_030_D4	Р	28064	51	0	0	216	194	1476	-123.6	63.5	638	34.83	19.1	864	47.1
Kar_1968_031_D5	Р	28064	51	0	0	216	155	1476	-117.0	63.5	756	30.76	19.1	756	44.7
Kar_1968_033_D7	Р	28064	51	0	0	216	194	1476	-145.5	63.5	752	34.48	19.1	864	47.6
Kar_1968_034_D8	Р	28064	51	0	0	229	194	1476	-142.8	76.2	738	34.83	19.1	1080	42.1
Kar_1968_035_D9	Р	27016	51	0	0	229	194	1476	-106.5	76.2	550	34.83	19.1	1364	29.2
Mahgoub_1975_002_A2	Р	33438	75	0	0	260	192	1560	-225.7	42.9	838	38.58	10.0	930	78.2
Mahgoub_1975_014_A7-1	P	33438	75	0	0	260	192	1560	-251.5	42.9	933	36.24	10.0	795	87.8
Mahgoub_1975_015_A7-2	Р	33438	75	0	0	260	192	1560	-251.5	42.9	933	36.24	10.0	770	88.2
Mahgoub_1975_017_A8-2	Р	33438	75	0	0	260	192	1560	-248.4	42.9	922	31.05	10.0	770	75.9
Mahgoub_1975_022_A11-2	Р	33438	75	0	0	260	192	1560	-179.9	42.9	668	29.06	10.0	905	64.9
Mahgoub_1975_025_B2	P	29250	50	0	0	260	192	1560	-206.8	42.9	768	35.88	10.0	1060	49.5
Mahgoub_1975_039_B11-2	P	29250	50	0	0	260	192	1560	-229.9	42.9	853	38.01	10.0	705	62.6
Mahgoub_1975_012_C12	P	38438	75	0	0	260	192	1560	-200.3	42.9	743	24.87	10.0	1060	53.6
Mahgoub_1975_012_C17	P	38438	75	0	0	225	245	1570	-98.0	75.0	399	26.43	10.0	1325	41.5
Mahgoub_1975_012_C19	P	38438	75	0	0	260	245	1570	-94.2	75.0	384	36.24	10.0	1180	55.3
Mahgoub_1975_015_D5	P	35250	50	0	0	260	192	1560	-187.2	42.9	695	31.97	10.0	1060	44.6
Mahgoub_1975_015_D6	P	35250	50	0	0	260	192	1560	-188.6	42.9	700	38.44	10.0	795	52.1
Mahgoub_1975_015_D8	P	35250	50	0	0	260	245	1570	-95.9	75.0	391	32.54	10.0	1325	27.3
Mahgoub_1975_015_D9	P	35250	50	0	0	260	245	1570	-95.2	75.0	388	36.24	10.0	1180	36.8
Mahgoub_1975_016_E1-1	P	43438	75	0	0	260	192	1560	-190.3	42.9	706	45.54	10.0	795	77.0
Mahgoub_1975_016_E1-2	P	43438	75	0	0	260	192	1560	-190.3	42.9	706	45.54	10.0	795	72.9
Mahgoub_1975_019_E4	P	43438	75	0	0	260	192	1560	-225.0	42.9	835	35.53	10.0	1060	58.8
Mahgoub_1975_019_E5-1	P	43438	75	0	0	260	192	1560	-245.9	42.9	913	37.73	10.0	1060	69.9
Mahgoub 1975 020 E6	P	43438	75	0	0	260	192	1560	-274.5	42.9	1019	35.53	10.0	795	93.4
Mahgoub_1975_021_F2	P	41250	50	0	0	260	192	1560	-193.5	42.9	718	35.03	10.0	1060	41.9
Olesen 1967 003 B1434	P	35039	79	0	0	262	132	1572	-92.6	109.2	793	19.76	9.5	914	41.3
Olesen_1967_004_B1441	P	34639	76	0	0	254	156	1572	-122.7	103.2	786	20.31	9.5	914	44.5
Sozen_1959_001_A1143	R	46452	152	0	0	209	284	1434	-122.7	56.9	800	42.90	38.1	1321	54.8
Sozen 1959 002 A1151	R	46452	152	0	0	203	161	1503	-126.3	62.0	786	20.00	38.1	1321	31.6
Sozen 1959 003 A1153	R	46452	152	0	0	204	241	1503	-206.6	51.3	858	30.07	38.1	1321	42.2
Sozen_1959_005_A1223	R	47226	152	0	0	204	161	1503	-126.4	84.6	787	38.97	38.1	914	60.8
Sozen_1959_006_A1231	R	46452	155	0	0	219	201	1503	-120.4	67.1	786	40.00	9.5	914	60.1
Sozen_1959_006_A1231 Sozen 1959 007 A1234	R	46452	152	0	0	219	201	1434		55.9	758		38.1	914	74.4
Sozen_1959_007_A1234 Sozen_1959_008_A1236		40452	152	0	0	208			-215.3			55.10 23.72		914	48.9
	R				0		150	1421	-117.5	81.0	785		38.1		
Sozen_1959_009_A1242	R	46452	152	0	0	211	284	1434	-202.4	58.4	713	43.17	38.1	914	70.0
Sozen_1959_010_A1246	R R	46452	152	0	0	208 218	227	1434	-205.7	55.9	906	32.14	38.1	914 914	63.1
Sozen_1959_012_A1253		46452	152		-		201	1503	-149.8	66.0	747	23.45	9.5	-	54.8
Sozen_1959_013_A1256	R	46452	152	0	0	218	234	1472	-194.0	65.8	831	26.14	9.5	914	59.7
Sozen_1959_018_A1439	R	46452	152	0	0	212	141	1503	-113.5	59.7	807	23.10	38.1	610	65.2
Sozen_1959_019_A1444	R	46452	152	0	0	216	161	1503	-130.7	63.5	814	23.10	38.1	610	72.0
Sozen_1959_020_A1455	R	46452	152	0	0	217	201	1503	-161.8	64.3	807	22.90	38.1	610	81.5
Sozen_1959_022_A2129	R	46452	152	0	0	215	101	1503	-42.4	62.2	421	23.10	38.1	1321	18.6

	Concrete cross section			Reinfor	cement			Pres	Сог	ıcrete	LOAD				
Notation	Туре	A _c	b _w	d _s	A _s	d _p	A _p	f _{py}	Р	e _p	$\sigma_{\rm p}$	f _c	D _{lower}	a	V _{test}
		mm ²	mm	mm	mm ²	mm	mm ²	MPa	kN	mm	MPa	MPa	mm	mm	kN
Sozen_1959_024_A2151	R	46452	152	0	0	206	301	1503	-122.8	53.8	407	38.83	38.1	1321	38.9
Sozen_1959_025_A2220	R	46452	152	0	0	215	114	1434	-47.9	62.2	422	36.90	38.1	914	33.2
Sozen_1959_026_A2224	R	46452	152	0	0	224	95	1434	-38.5	71.1	406	23.93	38.1	914	32.2
Sozen_1959_028_A2227	R	46452	152	0	0	213	114	1434	-47.0	60.5	414	26.55	38.1	914	31.8
Sozen_1959_029_A2228	R	47226	155	0	0	222	114	1434	-38.6	69.9	340	24.00	38.1	914	29.7
Sozen_1959_030_A2231	R	46452	152	0	0	205	114	1434	-70.0	52.3	616	24.34	38.1	914	34.2
Sozen_1959_031_A2234	R	46452	152	0	0	212	151	1434	-61.4	59.5	407	28.62	38.1	914	31.6
Sozen_1959_032_A2236	R	46452	152	0	0	212	114	1434	-68.9	59.7	607	19.93	38.1	914	33.7
Sozen_1959_033_A2239	R	46452	152	0	0	224	114	1434	-28.3	71.1	249	17.79	38.1	914	24.8
Sozen_1959_034_A2240	R	46452	152	0	0	208	246	1434	-122.0	55.9	496	39.93	38.1	914	59.7
Sozen_1959_035_A2249	R	46452	152	0	0	208	246	1434	-96.3	55.9	392	32.83	38.1	914	52.0
Sozen_1959_036_A3222	R	46452	152	0	0	238	114	1434	-18.8	85.9	165	29.59	38.1	914	32.2
Sozen_1959_037_A3227	R	46452	152	0	0	233	114	1434	-7.8	80.3	69	19.31	38.1	914	28.8
Sozen_1959_038_A3237	R	46452	152	0	0	208	246	1434	-8.5	55.9	34	42.21	38.1	914	40.0
Sozen_1959_039_A3249	R	46452	152	0	0	208	246	1434	-57.6	55.9	234	32.83	38.1	914	47.5
Sozen_1959_041_B1120	Р	33952	75	0	0	259	115	1628	-97.8	106.9	851	31.21	9.5	1321	31.0
Sozen_1959_042_B1129	Р	34064	75	0	0	254	154	1372	-131.8	101.6	855	28.90	9.5	1321	38.7
Sozen_1959_043_B1140	Р	34064	75	0	0	254	232	1372	-186.8	101.6	807	31.03	9.5	1321	46.6
Sozen_1959_045_B1210	Р	34692	78	0	0	282	78	1472	-66.2	129.8	848	38.62	9.5	914	35.8
Sozen_1959_049_B1226	Р	35095	77	0	0	256	150	1462	-114.0	103.1	758	30.76	9.5	914	52.6
Sozen_1959_050_B1229	Р	34452	76	0	0	248	154	1628	-128.8	95.5	839	28.83	9.5	914	56.6
Sozen_1959_051_B1234	Р	35483	78	0	0	259	225	1462	-166.7	106.2	740	33.28	9.5	914	64.6
Sozen_1959_052_B1235	Р	35894	78	0	0	254	154	1628	-128.1	101.3	834	22.14	9.5	914	51.5
Sozen_1959_053_B1250	Р	34292	75	0	0	259	193	1372	-154.3	106.7	800	20.34	9.5	914	51.6
Sozen_1959_054_B1261	Р	34452	76	0	0	251	232	1372	-182.8	99.1	789	20.55	9.5	914	53.8
Sozen_1959_056_B1316	Р	34452	76	0	0	264	115	1372	-99.9	111.3	865	38.21	9.5	711	59.6
Sozen_1959_057_B1326	Р	34212	75	0	0	255	154	1372	-131.8	102.4	855	31.72	9.5	711	65.0
Sozen_1959_058_B1341	Р	34052	74	0	0	255	232	1372	-189.2	102.6	817	29.79	9.5	711	71.2
Sozen_1959_059_B2126	Р	34292	75	0	0	259	154	1628	-66.0	106.9	430	30.83	9.5	1321	27.9
Sozen_1959_060_B2209	Р	34292	75	0	0	281	77	1628	-33.6	128.8	438	43.59	9.5	914	32.1
Sozen_1959_061_B2223	Р	34639	76	0	0	255	154	1628	-58.5	102.4	381	35.31	9.5	914	42.1
Sozen_1959_062_B2230	Р	35453	79	0	0	258	113	1462	-44.1	105.4	391	19.10	9.5	914	34.1
Sozen_1959_063_B2241	Р	36027	80	0	0	255	150	1462	-53.1	102.1	353	18.69	9.5	914	39.5
Sozen_1959_065_B2268	Р	34452	76	0	0	251	232	1372	-94.2	99.1	407	18.41	9.5	914	42.7
PWRI_1995_001_H3-35-30	R	85000	200	0	0	350	837	1799	-136.4	45.8	122	89.11	20.0	1050	247.0
PWRI_1995_002_H3-35-60	R	85000	200	0	0	350	837	1799	-272.8	45.8	244	83.60	20.0	1050	285.3
PWRI_1995_003_H3-35-90	R	85000	200	0	0	350	837	1799	-409.2	45.8	367	68.12	20.0	1050	296.9
PWRI_1995_004_H3-55-30	R	125000	200	0	0	550	837	1799	-193.5	79.2	173	81.32	20.0	1650	231.4
PWRI_1995_005_H3-55-60	R	125000	200	0	0	550	837	1799	-387.1	79.2	347	75.81	20.0	1650	364.5
PWRI_1995_006_H3-75-30	R	165000	200	0	0	750	837	1799	-251.7	112.5	226	85.69	20.0	2250	351.3
PWRI_1995_007_H3-75-60	R	165000	200	0	0	750	837	1799	-503.4	112.5	451	84.65	20.0	2250	442.2
PWRI_1995_008_H3-95-60	R	220000	200	0	0	1025	2093	1778	-728.1	131.9	307	69.16	20.0	2850	595.8
PWRI_1995_010_L3-35-30	R	85000	200	0	0	350	837	1778	-136.4	45.8	122	48.17	20.0	1050	195.4
PWRI_1995_011_L3-35-60	R	85000	200	0	0	350	837	1778	-272.8	45.8	244	39.33	20.0	1050	203.3
Funakoshi_1981_006_10	Р	23800	70	200	121	160	400	1226	-294.4	47.3	736	83.22	15.0	416	148.4
Funakoshi_1981_008_14	Р	23800	70	200	121	160	400	1226	-259.0	47.3	648	56.53	15.0	417	120.5
 Funakoshi_1981_009_19	Р	23800	70	200	121	160	400	1226	-259.0	47.3	648	56.05	15.0	504	96.9
Funakoshi_1982_006_38	Р	23800	70	200	121	160	390	1236	-211.9	47.3	543	41.52	15.0	414	96.9
Funakoshi_1982_007_39	P	23800	70	200	121	160	390	1236	-211.9	47.3	543	40.38	15.0	499	81.2
Funakoshi_1982_008_40	P	23800	70	200	121	160	390	1236	-211.9	47.3	543	40.38	15.0	583	68.9

	Concr	ete cross s	ection	Reinfor	cement			Pres	tressing			Concrete		LOAD	
Notation	Туре	A	b _w	d	A	d _p	A _p	f _{py}	Р	e _p	$\sigma_{\rm p}$	f	D	a	V _{test}
		mm ²	mm	mm	mm ²	mm	mm ²	MPa	kN	mm	MPa	MPa	mm	mm	kN
Sato_1987_001_3-4	Р	90000	150	375	860	330	804	1010	-294.3	180.0	366	40.47	15.0	1080	163.7
Sato_1987_004_3-7	Р	90000	150	375	860	330	804	1010	-490.5	180.0	610	41.04	15.0	1080	168.6
Sato_1987_007_3-11	R	67500	150	375	860	330	804	1010	-196.2	105.0	244	40.47	15.0	990	172.2
Sato_1987_009_3-13	R	67500	150	0	0	330	804	1010	-392.4	105.0	488	36.39	15.0	1080	159.3
Sato_1987_011_4-6	R	60000	150	375	142	330	804	1010	-294.3	130.0	366	38.86	15.0	1080	171.0
Sato_1987_012_4-7	R	60000	150	0	0	330	804	1010	-294.3	130.0	366	42.18	15.0	1080	168.0
Sato_1987_013_4-10	R	60000	150	375	860	330	804	1010	-98.1	130.0	122	39.62	15.0	1080	100.3
Sato_1987_014_4-11	R	60000	150	375	860	330	804	1010	-196.2	130.0	244	41.04	15.0	1080	150.4
Sato_1987_015_4-12	R	60000	150	375	860	330	804	1010	-294.3	130.0	366	44.18	15.0	1080	163.1
Ito_1996_001_NC-50	R	45000	150	275	143	250	227	1197	-33.1	100.0	146	54.06	15.0	650	117.8
Ito_1996_002_NC-100	R	45000	150	275	143	250	227	1197	-111.6	100.0	491	54.06	15.0	650	119.3
Ito_1997_001_M-B 100	Р	62000	100	400	253	270	454	1158	-213.7	111.6	471	40.19	20.0	1200	124.8
Saqan_2009_003_V-4-2.37	R	265187	373	660	1529	610	395	1676	-480.4	254.0	1217	53.45	19.1	2019	373.1
Saqan_2009_005_V-7-1.84	R	257419	362	664	1187	610	691	1676	-490.6	254.0	710	53.10	19.1	2019	490.8
Saqan_2009_006_V-7-2.37	R	252903	356	660	1529	610	691	1676	-490.6	254.0	710	53.10	19.1	2019	433.7
Saqan_2009_007_V-10-0	R	257419	362	0	0	610	987	1676	-494.2	254.0	501	51.72	19.1	2019	412.1
Saqan_2009_008_V-10-1.51	R	257419	362	663	974	610	987	1676	-494.2	254.0	501	51.72	19.1	2019	446.7
Saqan_2009_009_V-10-2.37	R	257419	362	660	1529	610	987	1676	-494.2	254.0	501	51.72	19.1	2019	446.7
Choulli_2007_S1E	Р	188900	100	0	0	671	1773	1776	-1805.9	243.4	891	99.15	12.0	2091	503.8
Choulli_2007_S1W	Р	188900	100	0	0	671	1773	1776	-1805.9	243.4	891	99.15	12.0	2087	521.7
Choulli_2007_S2E	Р	188900	100	0	0	700	1013	1776	-1205.2	216.5	951	96.34	12.0	2091	366.8
Choulli_2007_S2W	Р	188900	100	0	0	700	1013	1776	-1205.2	216.5	951	96.34	12.0	2087	368.7
Zink_2000_SV-2	R	70000	175	350	1256	350	140	1570	-274.4	75.0	980	105.28	11.0	1225	178.9
Zink_2000_SV-4	R	140000	350	357	1963	345	560	1570	-1086.4	72.5	970	93.67	16.0	1225	509.7
Zink_2000_SV-5	R	280000	350	763	402	733	1680	1570	-3225.6	166.5	960	89.53	16.0	2600	736.5
Wilder_2014_B103	Р	84670	70	511	0	511	744	1737	-1261.8	141.9	1488	77.50	12.0	2000	262.8
Wilder_2014_B106	Р	84670	70	511	0	511	744	1737	-636.0	141.9	750	88.90	12.0	2000	179.7
Wilder_2014_B109	Р	84670	70	550	0	550	373	1737	-710.4	130.4	1488	89.30	12.0	2000	181.0
Joergensen 2021 PB5-750A	R	175000	250	641	942	500	1050	1560	-750.0	150.0	714	58.50	16.0	3500	200.9
Joergensen 2021 PB5-750B	R	175000	250	641	942	500	1050	1560	-750.0	150.0	714	60.70	16.0	3500	189.2
Joergensen 2021 PB5-1250A	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	56.30	16.0	3500	238.4
Joergensen 2021 PB5-1250B	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	57.30	16.0	3500	230.7
Joergensen 2021 PB4-1250A	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	60.60	16.0	2800	290.9
Joergensen 2021 PB4-1250B	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	62.50	16.0	2800	301.1
Joergensen 2021 PB6-1250A	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	59.40	16.0	4200	219.9
Joergensen 2021 PB6-1250B	R	175000	250	641	942	500	1050	1560	-1250.0	150.0	1190	60.20	16.0	4200	212.3

Type: P (profiled section); R (rectangular section)

cross-sectional area of concrete

minimum width of the cross-section between tension and compression chords

distance from outermost compressed fibre of the cross-section to the centroid of the ordinary reinforcement

cross-sectional area of the longitudinal ordinary reinforcement located in the tensile zone due to bending

distance from outermost compressed fibre of the cross-section to the centroid of the prestressed reinforcement

cross-sectional area of the longitudinal prestressed reinforcement located in the tensile zone due to bending

 $\begin{array}{l} A_{c} \\ b_{w} \\ d_{s} \\ d_{s} \\ A_{s} \\ d_{p} \\ A_{p} \\ f_{py} \\ P \\ P \\ \sigma_{p} \\ f_{c} \\ f_{p} \\ \sigma_{p} \\ f_{c} \\ D_{lower} \end{array}$ yield strength of prestressing steel

prestressing force

eccentricity of the prestressing force related to the centroid of the cross-section, positive when the eccentricity is on the side of the flexural reinforcement in tension

stress in prestressing steel

cylinder compressive strength of concrete

Smallest value of the upper sieve size in an aggregate for the coarsest fraction of aggregates

shear span a:

V_{test}: shear force at failure