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A Mechanical Approach for the Punching Shear Provisions in the Second Generation of Eurocode 2

El tratamiento del punzonamiento en la segunda generación del Eurocódigo 2 basado en un modelo mecánico

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ABSTRACT

The revision of a code is a long-term project that shall fulfil several aims, comprising the enhancement of the ease-of-use and incorporating updated state-of-the-art. With respect to the revision of Eurocode 2 concerning the punching shear provisions, this task allowed also for the opportunity to enhance the understanding of the code and physical phenomenon by designers. The original EN1992-1-1:2004 punching provisions were adapted from an empirical equation for design based on the regression analyses performed by Zsutty in the 1960s for shear in beams and later reworked in Model Code 1990 for punching shear. These expressions did not show any link to the physical response of a structure, making difficult to designers to clearly understand how to engineer their designs. Instead of continuing with this approach, CEN/TC250/WG1 took the decision in 2016 to ground the punching provisions on a mechanical model that could be explained to engineers, allowing for a transparent understanding of the design equations and phenomena. To that aim, the Critical Shear Crack Theory, already implemented in Model code 2010 at that time, was selected as representative of the state-of-the-art. Following that decision, a large effort has been performed to implement this theory into the Eurocode, keeping its simplicity of use and generality. This paper is aimed at presenting the theoretical grounds of the theory as well as the manner in which it is drafted for the future generation of Eurocode 2.

KEYWORDS: Punching, shear, reinforced concrete, slabs, column bases, mechanical model.

RESUMEN

La revisión de una norma es un proyecto que debe cumplir varios objetivos, entre los que se encuentran la mejora de la facilidad de uso y la consideración del estado del conocimiento más avanzado y robusto. En el marco de la revisión de las disposiciones relativas al punzonamiento de placas para el futuro Eurocódigo 2, se dedicaron también amplios esfuerzos para mejorar la coherencia entre las expresiones de diseño y los fenómenos físicos asociados, con el objetivo de facilitar la comprensión de la norma por los proyectistas. Debe observarse que las expresiones originales de la norma EN1992-1-1:2004 se desarrollaron a partir de una ecuación empírica para el diseño de vigas a cortante basada en los trabajos de Zsutty en la década de 1960. Dicha expresión fue posteriormente modificada e introducida en el Código Modelo de 1990 para la resistencia a punzonamiento. Estas expresiones empíricas, a pesar de ser sencillas de aplicar, no permiten comprender la respuesta mecánica de una estructura ni los mecanismos físicos que llevan a su fallo a punzonamiento. Esta pérdida de conexión con la física del fenómeno dificulta a los proyectistas comprender de manera clara cómo mejorar sus diseños o cuestiona la aplicación de las expresiones fuera de los rangos en los que han sido calibradas. En lugar de continuar con un enfoque empírico, el CEN/TC250/WG1 tomó la decisión en 2016 de basar las disposiciones para el punzonamiento de placas en un modelo mecánico que pudiera ser explicado a los ingenieros. Para ello, se seleccionó la Teoría de la Fisura Crítica, implementada previamente en el Código Modelo 2010 como referente del estado del conocimiento. Tras esa decisión, la implementación de la Teoría de la Fisura Crítica en el Eurocódigo ha requerido diversas consideraciones específicas con el objetivo de mantener el formato del Eurocódigo 2 pero respetando la simplicidad de uso y generalidad de la teoría. Este artículo presenta así los fundamentos de la Teoría de la Fisura Crítica, así como la manera en que se ha incorporado en la futura generación del Eurocódigo 2.

PALABRAS CLAVE: Punzonamiento, cortante, hormigón armado, placas, conexiones columna-placa, modelo mecánico.

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1. INTRODUCTION

Punching shear is a brittle failure mode associated to the penetration of a loaded area in a concrete slab. Slab-column connections where punching occurs experience in general a sudden loss of load-carrying capacity and this can trigger punching failures at other regions (which is a typical situation in flat slabs supported by columns) leading to a progressive collapse [1]. Despite being a phenomenon well-known and having attracted many research efforts in the past [2-6], scanty approaches have been developed so far to lead to physically-based design approaches. As a consequence, design has been traditionally performed on the basis of empirical equations [7-10]. For instance, Eurocode 2 (EN1992-1-1:2004) [8] based its punching design formulation on the approach of one-way slabs and beams failing in shear by adopting the empirical formulation established by Zsutty in 1968 [11]. Such approach had the advantage of keeping a consistent unitary resistance for both verifications (one- and two-way shear). However, it required to define a control perimeter relatively far from the supported area (typically the column) and not linked to the mechanics of the phenomenon. Also, adapting the empirical formulation to other cases that were observed to be relevant from a practical point of view and whose different response was confirmed experimentally (for instance different yield strength of the reinforcement or considering the influence of the slenderness) was not possible. The engineer had, in fact, little help from the empirical formula on the physics of the phenomenon and how to design suitably and in a robust manner.

With respect to physical design models, Kinnunen and Nylander in Sweden proposed in the 1960s [12] an approach that constituted a significant advancement in the understanding of the phenomenon and its prediction. This model considered shear to be carried by a conical strut whose failure in compression leads to the punching failure of the slab-column connection. According to Kinnunen and Nylander [12], failure was assumed to occur for a given level of the compressive tangential strain developing in the soffit of the slab in vicinity of the supported area. By adopting a kinematics defined by a conical deformation in the outer region of the slab, a failure criterion was established as a function of the rotation of the slab. The theory of Kinnunen and Nylander [12] allowed for relatively accurate predictions and was later adapted by other researchers and extended to a number of cases. Amongst these approaches consistent with the principles established by Kinnunen and Nylander [12], a theory named the Critical Shear Crack Theory (CSCT) was developed in 1985 [13] for the Swiss code for structural concrete SIA 162 [14] and later elaborated by Muttoni and Schwartz in 1991 [15]. Originally developed for punching of slab-column connections without shear reinforcement, it was later extended to one-way slabs failing in shear [16] and also to punching of connections with shear reinforcement [17], prestressing [18, 19], footings [20], fibre reinforced concrete [21] and slabs strengthened with post-installed shear reinforcement [22] or fibre reinforced polymers [23].

The formulation of the CSCT allowed to be implemented into design codes following simple design expressions, showing in a transparent manner the various parameters implied in the phenomenon. These parameters can be evaluated in simple and safe manners or following accurate analyses. On that basis, the CSCT was successfully formulated in terms of a Levels-of-Approximation (LoA) approach [24] and implemented into *fib* Model Code 2010 [25], constituting a significant advance with respect to previous approaches. The LoA approach is a design philosophy [24,26] very much aligned to engineering daily practice. Safe and simple estimates are first performed with a limited amount of work, whose accuracy can be refined upon necessity, requiring some additional work to better evaluate the parameters. For the punching formulation of the CSCT in fib Model Code 2010 [25], these levels were structured as follows:

- LoA I: Aimed at a preliminary check and identification of potentially-critical regions. When the resistance of this level is satisfied, bending and not punching is expected to govern the design.
- LoA II: Aimed at typical design for a slab-column connection failing in punching. Implemented by means of analytical formulae exclusively.
- LoA III: Refinement of the previous level, by evaluating several physical parameters on the basis of a linear-elastic finite element analysis. This level is only intended for design of unusual cases or for assessment of existing structures.
- LoA IV: Procedure considering both the failure criterion of the CSCT for the resistance and a demand curve (load-rotation relationship) established on the basis of a nonlinear flexural analysis of the slab. Such level is the most accurate prediction. It allows considering in a consistent manner a number of effects traditionally neglected for design (such as membrane action) but is relatively time-consuming and intended mostly for the assessment of critical cases.

The *fib* Model Code 2010 [25] constituted a significant advancement in design procedures. The expressions for punching design were also checked in numerous scientific works performed all over the world, verifying its accuracy or helping to refine it. The formulation of the *fib* Model Code 2010 is also very practical, allowing for a direct design procedure [26] by verifying that the design value of the resistance V_{Rd} is not lower than the design shear force V_{Ed}. For the explicit calculation of the punching resistance, performing an iterative procedure (intersection of failure criterion and load-rotation relationship) is however required.

Within the revision of Eurocode 2, several deficiencies of the empirical design formula of EN1992-1-1:2004 [8] were highlighted [e.g. 28-37] as well as the limitations of the approach in terms of generality to address relevant topics, such as new materials, unusual geometries or the assessment of existing structures. In addition, the necessity to move to more rational and physically-based models was early identified. After careful analysis of potential approaches, the CSCT was eventually selected as the basic model for the new provisions concerning punching shear. The physical grounds of the CSCT and its flexibility for implementation into design expressions were largely appreciated. However, a relatively direct transcription of the fib Model Code 2010 [25] was not considered appropriate within the Eurocode 2 design philosophy. Thus, it was decided to implement the CSCT following a different approach:

- The preliminary check, identifying regions where shear failures are not expected to be governing was implemented in the definition of the minimum shear resistance.
- The general procedure for punching shear verification had to be based on analytical formulae. This has analogies with the LoA II of Model Code 2010, but its design expressions shall be written a closed-form manner, avoiding iterative procedures for calculation of the punching resistance
- The Annex for assessment of existing structures (Annex I, of informative nature in FprEN1992-1-1:2022 [42]) could elaborate more detailed solutions, comprising the results of nonlinear analyses and verification procedures

With this task in mind, the Task Group 4 (TG4; "Shear, Punching and Torsion") of CEN/TC250/SC2/WG1 tailored the formulation of the CSCT to the needs of the Eurocode 2. This task required efforts in a number of fields, from the definition of the failure criterion to the verification of the level of safety of the design expressions. The current provisions FprEN1992-1-1:2022 [42] reflect the work performed during the last seven years incorporating the comments and advices of the various participants of CEN.

In this paper, a review of the EN1992-1-1:2004 [8] is first presented, showing also the reasons for change. Then, the theoretical principles of the CSCT are introduced. The main formulae and simplifications introduced for the derivation of the closed-form expressions of FprEN1992-1-1:2022 [42] are also shown and justified. Finally, a practical example is presented, showing the simplicity of the approach and its generality.

2. PUNCHING DESIGN ACCORDING TO FIRST GENERATION OF EUROCODE 2 (EN 1992-1-1:2004)

2.1 Code formulation

The design approach of current Eurocode 2 (EN 1992-1-1:2004) [8] with respect to punching is formulated in terms of closed-form equations, where the action and resistance are evaluated on the basis of a number of geometrical and mechanical parameters. The different formulae have in fact an empirical nature. Namely, the one referring to design of members without shear reinforcement can be considered as a direct adaption of the works of Zsutty [11] for the shear resistance of beams to the punching resistance of two-way slabs:

$$v_{Rd,c} = C_{Rd,c}\, k\, (100\, \rho_l\, f_{ck})^{1/3} + k_1\, \sigma_{cp} \geq (v_{min} + k_1\, \sigma_{cp}) \ \ [\text{MPa}] \quad (1)$$

where

- $C_{Rd,c}$, v_{min} and k_1 are NDPs, whose proposed values are $C_{Rd,c} = 0.18/\gamma_C$, $v_{min} = 0.035\, k^{3/2}\, f_{ck}^{1/2}$ and $k_1 = 0.1$.
- $k = 1+\sqrt{\frac{200}{d}} \leq 2.0$ with d in [mm] is the factor accounting for the size effect
- f_{ck} is the cylinders characteristic concrete compressive strength in [MPa]
- $\rho_l = \sqrt{\rho_{l,y}\, \rho_{l,z}} \leq 0.02$ is the geometric mean of the steel reinforcement ratio relating to the bonded tension steel in *y*- and *z*- directions respectively (calculated on a band of width equal to 3*d* on each side of the column plus the column size)
- $\sigma_{cp} = \frac{\sigma_{cy} + \sigma_{cz}}{2}$, with σ_y and σ_{cz} being the normal concrete stresses [MPa] in the *y*- and *z*- directions (positive if compression)

As it can be noted, the original expression of Zsutty [11] has been somewhat adapted to include other relevant effects as the size effect (parameter *k*). Also, corrections were proposed to account for the influence of compression and tension forces (term σ_{cp}) [38]. In order to maintain a uniform approach with the verification for shear (based on the same unitary resistance), the location of the control perimeter was thus tailored to a distance equal to 2*d* from the edge of the column (for typical verifications corresponding to the previous formula).

The approach followed is in fact very much inspired on the formulation proposed in Model Code 1990 [7]. The formula had served during almost 20 years, with a format that is apparently simple and accounting for a limited number of parameters, which is convenient for design. Although no major criticism was raised on its simplicity for use for standard cases in flat slabs, several theoretical inconsistencies were raised and partly amended in the EN 1992-1-1:2004 corrigenda [40, 41]. Some of the most important critics, justifying an update of the provisions, are presented in the next section.

2.2. Criticism of EN 1992-1-1:2004 and reasons for change

The section dedicated to the punching shear design in current EN 1992-1-1:2004 [8] was one of the parts that received more systematic review comments in 2013 before starting the revision for the 2nd generation of Eurocodes. Many reasons supported an in-depth revision, mostly addressing scientific concerns (state-of-the-art) and ease-of-use [28-37]. Some relevant critics are summarized below:

- A different methodology is prescribed for the verification of punching shear resistance of flat slabs and footings. For flat slabs, the control section is located at a nominal distance 2*d*, lacking of physical meaning (control section too far away from the critical region where punching develops). For footings, the location of the control section is calculated by minimization of the resistance, requiring lengthy and unpractical analyses (even if the use of software and spreadsheets can simplify the calculations).
- The size effect law included in the EN1992-1-1:2004 [8] approach does not suitably describe the phenomena [37]. The size effect can in fact be severely underestimated for thick slabs (too small decrease on the unitary strength for increasingly larger sizes) and the formula does not comply to any reasonable size-effect law [37].
- The current approach does not consider any slenderness effect [28]. The level of strains (and corresponding crack widths) is governed by the flexural deformations in bending [28] which is in turn represented by the flexural reinforcement ratio. However, the same amount of flexural reinforcement can lead to different crack openings and

associated punching resistances for varying slenderness. This effect was already observed by empirical analysis of data [6] and also by theoretical reasoning [28], and named in many cases strain-effect.

- The level of safety when compared to available test data is not uniform with respect to the various parameters implied (and also between footings and slender slabs). This has been observed by analysis of large experimental programmes performed since the 2000s (see a detailed overview in [43]).

In addition to these reasons, which suggest deficiencies in the formulae used, there is still a more significant one. It relates to the generality of the approach and its potential to adapt to new situations. The Zsutty's formula is in fact of empirical nature, obtained by regression of parameters compared to test results (as honestly stated in the title of that paper [11]). Every parameter or physical phenomena that has not been calibrated into the original formula is not reflected and the designer has no orientation on how to address it. This fact, which could be limiting but perhaps sufficient for a new design following a number of restrictions (detailing rules), is however very unsuitable for the assessment of existing structures. For instance, the formula does not provide guidance on how to account for the influence of reinforcement with higher or lower yield strength than usually arranged. This can however be relevant for design of new structures (use of new materials) and particularly for assessment of existing ones (in many cases with lower-resistance reinforcement). A similar situation happens with respect to other parameters, such as aggregate size or even influence of level of load when strengthening is performed [75]. The loss of physical meaning does not allow the designer to understand the potential detrimental or favourable effects and how to account for them (which can unfortunately be seen as new patches or coefficients in the existing formula).

In order to overcome these difficulties, it was decided by CEN/TC250/SC2/WG1 to ground the punching shear design provisions on the basis of a mechanical model. This shall allow to transparently clarify the role of the different parameters and their influence in the punching design formulae and to show also the relationship between them (as for instance between size and strain effects). Such approach should also allow for a sufficient level of generality, so that it can be safely applied to both unusual design situations (enhancing the freedom of the designer) and for assessment of existing structures.

3. A MECHANICAL MODEL FOR THE SECOND GENERATION OF EUROCODE 2 – THE CRITICAL SHEAR CRACK THEORY

As previously mentioned, after detailed analysis of several state-of-the-art models, TG4 of CEN/TC250/SC2/WG1 decided to adopt the Critical Shear Crack Theory (CSCT) as the grounds for the new provisions for punching shear design of FprEN 1992-1-1:2022 [42]. The theoretical bases of the CSCT are briefly presented in this section. A detailed description can be consulted elsewhere [17, 28, 45-49]. In the following sections, the adaptions introduced to implement it into FprEN 1992-1-1:2022 [42] and to respect the format of the Eurocode will be discussed.

3.1. Members without shear reinforcement

Two-way slabs subjected to concentrated loading develop both cracking associated to radial and tangential bending moments. Due to the presence of shear forces, tangential cracks develop in an inclined manner and can disturb the inclined compression struts carrying shear [28]. One of these cracks is named as the Critical Shear Crack (CSC), being the one intercepting the compression strut near the supported area (shear-critical region).

The mechanical and geometrical properties of the CSC govern the punching resistance. It localizes the strains in the shear-critical region due to the strong gradient of bending moments and shear forces close to the concentrated action

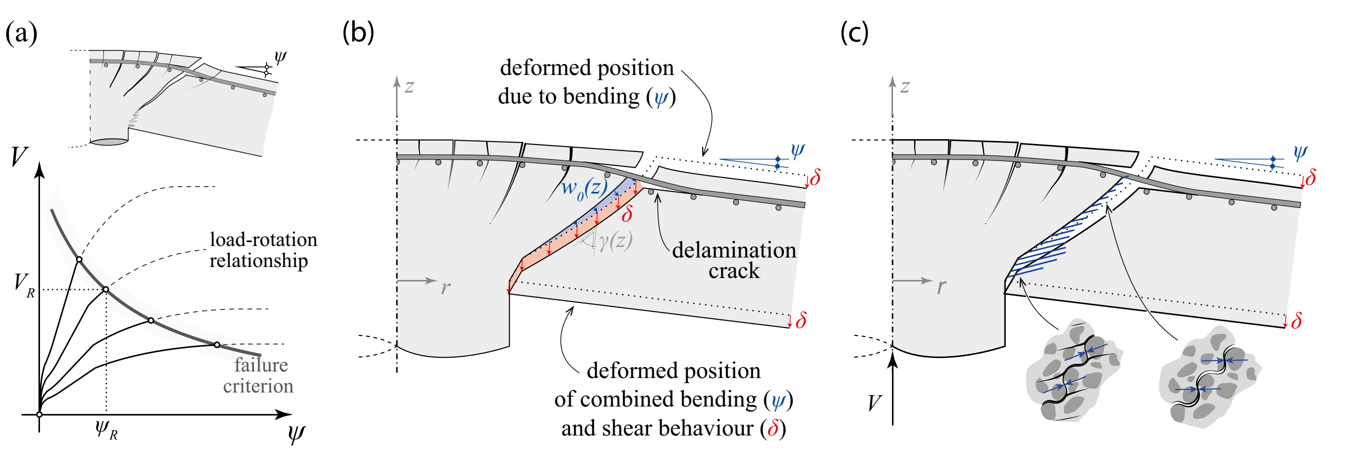

Figure 1. Theoretical principles of the mechanical model of Critical Shear Crack Theory (adapted from [46] and [49]): (a) intersection of load-rotation relationship and a failure criterion; (b) kinematics of the critical shear crack at failure; and (c) resulting internal stresses.

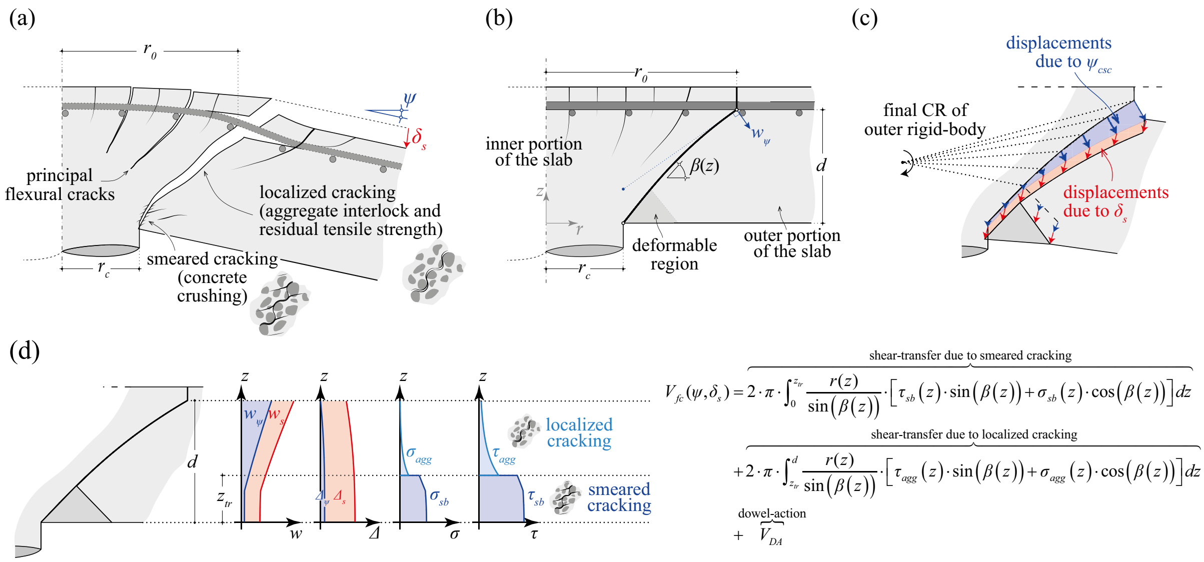

Figure 2. The refined mechanical model of Critical Shear Crack Theory according Simões et al. [48]: (a) observed behaviour: (b) adopted critical shear crack geometry; (c) adopted kinematics; (d) resulting normal and shear stresses (b4 and b5) (figures adapted from [48] and [49]).

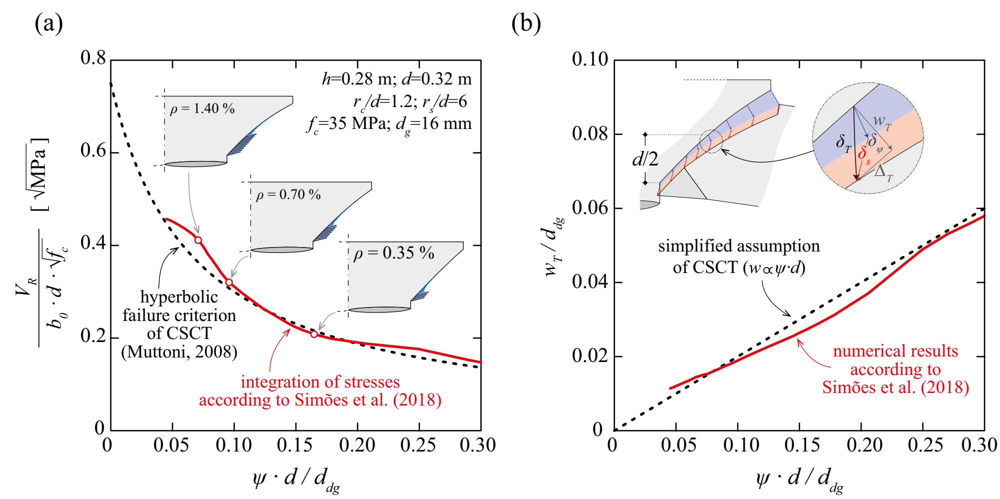

Figure 3. Results of mechanical model by Simões et al. [48] for an investigated case: (a) comparison of the numerically calculated failure criterion with the hyperbolic failure criterion (Eq. (2)) and representation of the resultants of stresses developing along the CSC for different rotations; and (b) numerically calculated crack opening–rotation relationship and comparison with simplified assumption of Muttoni [28].

[28, 47]. The CSC is usually originated at a distance close to one effective depth (d) and propagates in an inclined manner. Its opening is one of its key parameters for the punching resistance, as wider cracks reduce the ability of concrete to transfer shear stresses [51]. For slender members, such opening is mostly governed first by bending deformations but, when approaching to failure, shear deformations become also significant [52, 19, 46, 50, 48]. Eventually, at failure, the critical shear crack starts sliding leading to the development of the punching cone as shown in Figure 1 [52, 19, 50]. Based on these considerations, the Critical Shear Crack Theory (CSCT) considers that the kinematics of the CSC is composed by the sum of flexural (in blue in Figure 1b) and shear (in red in Figure 1b) movements. On that basis, the shear and normal stresses acting along the CSC can be calculated considering suitable material laws, see Figure 1c. This can be performed in a refined manner based on a numerical integration [52, 19, 48], see Figure 2.

Some results of the mechanical model of the advanced implementation of the CSCT [48] are shown in Figure 3a in terms of normalized punching resistance and normalized rotation. The results show a decrease on the punching resistance with increasing rotations of the slab. This is justified by the fact that larger rotations are associated to wider widths of the critical shear crack, thus reducing the contribution of the different shear-transfer actions (i.e. direct strutting, aggregate interlock, residual tensile strength and dowel action).

The results of this model are in fact in agreement with the model of Kinnunen and Nylander [12], considering the development of an inclined strut carrying shear near the column region (also called compressive cone; see stresses developing along the CSC in Figure 3a according to refined mechanical model of CSCT). Another interesting result can be obtained if the opening of the critical shear crack (accounting for both flexural and shear deformations) at a height $d/2$ from the slab soffit is represented as a function of the normalized rotation for the investigated case, refer to Figure 3b. The results show that the crack width and the normalized rotation are correlated and that a linear correlation is a fairly good approximation of it. It is interesting to note that Muttoni and Schwartz [15] suggested in 1991 such a linear relationship between the opening of the critical shear crack and the product $\psi\, d$ (linear correlation between crack width w and product of rotation $\psi$and effective depth d).

Other than the opening of the CSC, also its roughness influences the ability of the CSC to transfer shear forces [53, 54, 55]. In 2003, Muttoni [16] introduced this consideration by including the crack roughness, expressed in terms of the maximum aggregate size dg. Eventually, Muttoni [16, 28] proposed a hyperbolic failure criterion relating the punching resistance and the crack opening (represented by the product $\psi\, d$) and roughness (accounting for d_g) as follows [28]:

$$\frac{v_{r,c}}{b_{0.5}\, d_v} = \frac{0.75\sqrt{f_c}}{1+15\,\frac{\psi\, d}{d_{g0}+d_g}} \tag{2}$$

where $V_{R,c}$ refers to the punching shear resistance (concrete contribution); $b_{0,5}$ to the length of the control perimeter at a distance of $d_v/2$ from the column face (round corners in case of square or rectangular columns); d_v to the shear-resisting effective depth (potentially differing from the effective depth d to account for the penetration of the support and thus reducing the depth available to carry shear); f_c to the cylinders concrete compressive strength; d_{g0} to the reference aggregate size (d_{g0} = 16 mm for normal weight concrete).

Eq. (2) suitably represents the response of reinforced concrete slabs failing in punching when compared to available experimental results, see Figure 4a. The theory shows a decreasing punching strength for increasing level of rotations (according also to Kinnunen and Nylander [12]). The punching strength of a slab-column connection can therefore be obtained by intersecting the load-rotation relationship of the slab (defining the shear demand) and the failure criterion (representing the shear resistance associated to a state of deformations), refer to Figure 4b. One interesting aspect of the CSCT is that the load-rotation relationship of the slab can be calculated with different levels of refinement:

- Analytical formulae for axisymmetric cases based on the model by Kinnunen and Nylander [12]. These formulae were developed considering both a simplified bilinear moment-curvature relationship as well as more sophisticated laws accounting for tension-stiffening (quadri-linear laws) [15, 16, 28];
- Using a simplified formula for practical purposes, derived analytically from the general law [26]):

$$\psi = k_m\,\frac{a_p}{d}\,\frac{f_y}{E_s}\left(\frac{m_s}{m_R}\right)^{\frac{3}{2}} \tag{3}$$

  where a_p is the distance between the axis of the column and the line of zero radial moment, f_y and E_s are respectively the yield strength and the modulus of elasticity of the flexural reinforcement, k_m is a factor depending on the level of the refinement of the approach used to estimate the acting bending moment in the support width (typically 1.5 for simple analyses and 1.2 when some parameters are known more in detail), m_s is the acting bending moment in the support strip width (b_s) and m_R is the average moment capacity in the support width;
- Nonlinear finite element analyses, considering in detail geometrical and mechanical aspects of the response [56- 59].

The selection of the most suitable load-rotation relationship depends on many aspects, such as the level of knowledge of the structure. Very refined analyses, as those resulting from the application of the nonlinear finite element analyses are in principle only possible when the structure can be characterized in detail. This is the case when the resistance of a structure needs to be assessed, when the required geometric data, reinforcement and material properties can be finely evaluated. Otherwise, namely for design, simpler methods are preferable and more consistent with the degree of knowledge or definition of the structure.

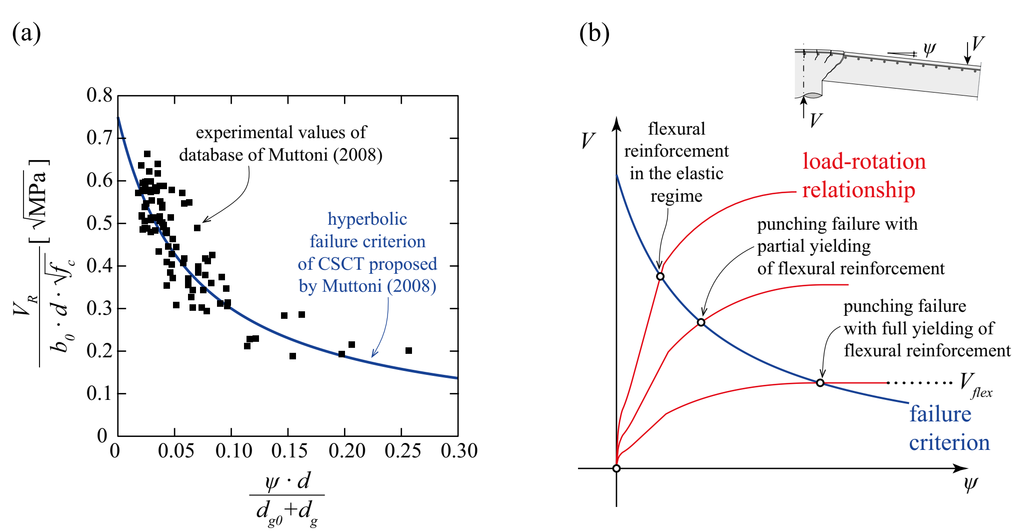

Figure 4. (a) Experimental validation of the hyperbolic failure criterion of CSCT (Eq. (2)) proposed by Muttoni in 2008 [28] and (b) potential punching shear failure regimes (figures adapted from [46]).

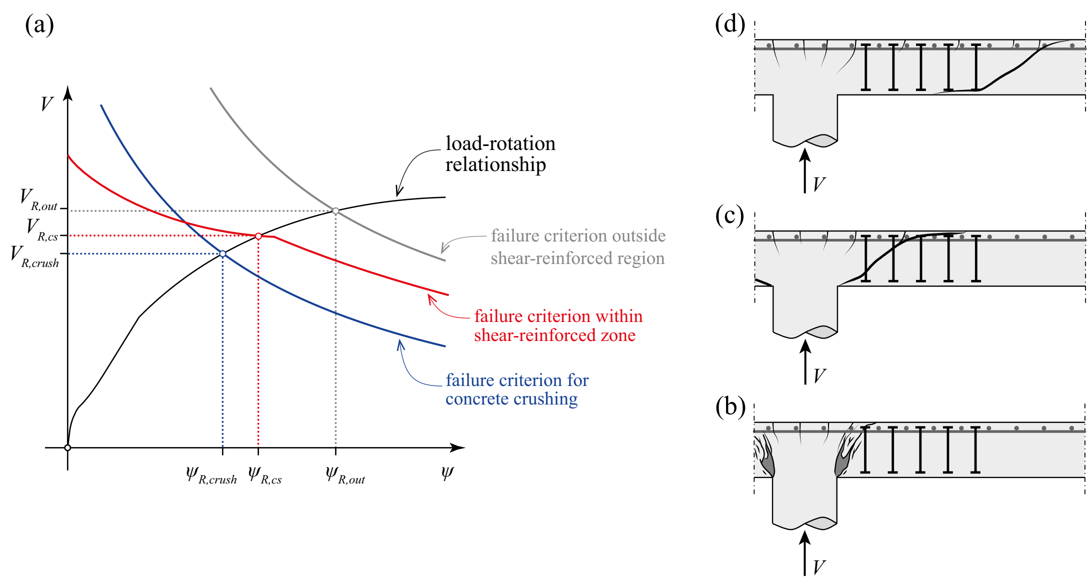

Figure 5. Members with shear reinforcement: (a) load-rotation relationship and intersection with possible governing failure criteria (Eqs. (4) to (7)) associated with (b) crushing of concrete struts, (c) failure within the shear-reinforced region; and (c) failure outside the shear-reinforced region (failure modes adapted from [17]).

3.2. *Members with shear reinforcement*

The arrangement of shear reinforcement is one of the most suitable solutions to enhance the resistance and deformation capacity of slabs [e.g. 60, 61, 17, 35]. The CSCT was extended consistently to this case maintaining its basic assumptions by Fernández Ruiz and Muttoni [17]. To that aim, when usual detailing rules are respected [25], three potential failure modes can govern [17]: (i) failure within the shear-reinforced area (Figure 5c); (ii) crushing of concrete struts (maximum punching strength; Figure 5b) and (iii) failure outside the shear reinforced area (Figure 5d). The approach proposed by the CSCT [17] allows calculating again the resistance by intersection of the load-rotation curve (assumed to be the same as for members without shear reinforcement, since the shear deformations are neglected) and the pertinent failure criterion for each of these modes.

Failure within the shear-reinforced area

As shown in Figure 6, the punching resistance can be calculated as the sum of concrete $V_{R,c,cs}$ and shear reinforcement $V_{R,s,cs}$ contributions [17]:

$$V_{R,cs} = V_{R,c,cs} + V_{R,s,cs} \tag{4}$$

In this Equation, the concrete contribution $V_{R,c,cs}$ is given by the failure criterion of the corresponding element without shear reinforcement (see Eq. (2)) and the shear reinforcement contribution ($V_{R,s,cs}$) is given for axisymmetric cases by:

$$V_{R,s,cs} = \sigma_{sw}\ \Sigma A_{sw} \leq f_{yw}\ \Sigma A_{sw} \tag{5}$$

where σ_{sw} is the average stress in the shear reinforcement intercepted by the punching cone (considered in a simplified manner to develop with an inclination of 45°), ΣA_{sw} is the total area

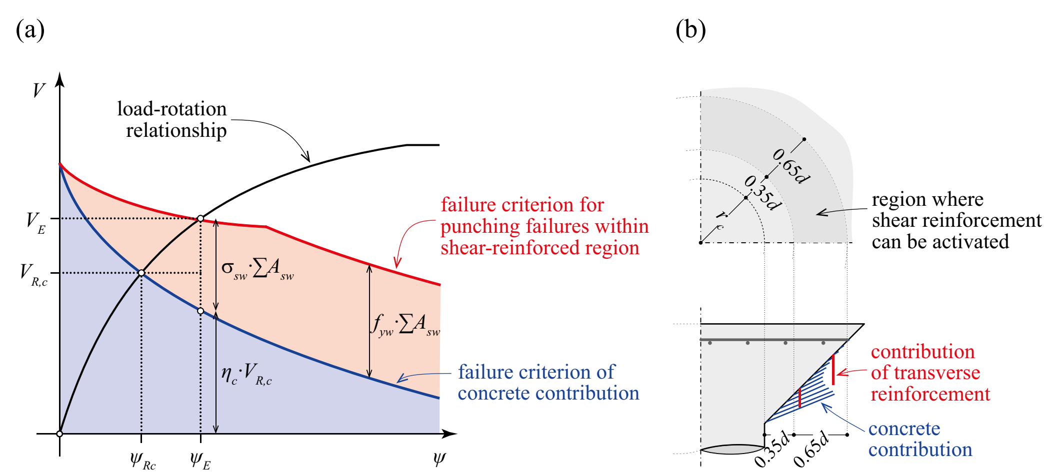

Figure 6. Definition of contributions of concrete and shear reinforcement for punching failure within the shear-reinforced region.

of the activated shear reinforcement in the punching cone (assumed within 0.35dv and dv) and f_{yw} is the yield strength of the shear reinforcement. The average stress in the shear reinforcement can be calculated as a function of the rotation, by assuming it to be proportional to the opening of the CSC plus a term accounting for bond (details accounting for advanced considerations on bond and anchorage can be consulted elsewhere [17]):

$$V_{sw} = \frac{E_{sw}}{6}\,\psi + f_b\,\frac{d_v}{\phi_w} \tag{6}$$

where f_b is the average bond stress, ϕ_w and E_{sw} are respectively the diameter and the modulus of elasticity of the shear reinforcement. For members governed by shear deformations (as footings or prestressed slabs) [62, 67], the maximum punching resistance can be associated to large shear deformations with the concrete contribution vanishing ($V_{R,c,c}\rightarrow 0$) and the stress in the shear reinforcement tending to the yield strength ($\sigma_{sw} \rightarrow f_{yw}$). For these cases, however, the extent where the punching shear reinforcement can be activated can be significantly reduced (steeper angle of the failure cone [29, 62, 63]).

Failure by crushing of concrete struts (maximum punching resistance)

Fernández Ruiz and Muttoni [17] proposed to evaluate the maximum punching resistance (crushing of concrete struts) as an enhancement of the punching strength of the corresponding element without shear reinforcement. This is justified by the fact that the crushing resistance of the concrete strut is, as for members without shear reinforcement, influenced by the opening of the CSC and by its roughness. In addition, it depends strongly on the anchorage conditions, geometry and detailing rules of the shear reinforcement [e.g. 17, 35, 60, 61, 64]. This condition can be expressed as:

$$V_{R,max} = \eta_{sys,sb}\, V_{R,c} \tag{7}$$

where $V_{R,max}$ is the punching resistance associated to crushing of the concrete struts, $V_{R,c}$ is the failure criterion of the corresponding slab-column connection without shear reinforcement (see Eq. (2)) and $\eta_{sys,sb}$ is an enhancement factor which depends on the type of shear reinforcement.

Failure outside the shear-reinforced area

To calculate the punching resistance of failures outside of the shear-reinforced area, Fernández Ruiz and Muttoni [17] considered in a safe manner that the rotations of the critical shear crack concentrate outside of the shear-reinforced area. This is equivalent to considering the shear-reinforced area as a stiff supported region. On that basis [17], the same failure criterion as for slabs without shear reinforcement can be used, provided a suitable value of the control perimeter is selected:

$$\frac{v_{R,out}}{b_{0.5,\mathrm{out}}\, d_{v,out}} = \frac{0.75\sqrt{f_c}}{1+15\,\frac{\psi\, d}{d_{g0}+d_g}} \tag{8}$$

where $dv_{,out}$ is the shear-resisting effective depth of the outer perimeter of reinforcement (see Figure 5d). On that basis, $b_{0,5,out}$ is the outer control perimeter (defined at $0.5d_{v,out}$ from the last perimeter of shear reinforcement and accounting for some limitations in the distances between the shear reinforcement units).

3.3. *Considerations for eccentric punching*

The development of a non-uniform distribution of shear forces along the control perimeter in the cases of eccentric punching (internal columns with unbalanced moments, presence of large openings in the vicinity of columns, edge and corner columns) is accounted in the framework of the Critical Shear Crack Theory by setting two different control perimeters (as defined in fib Model Code 2010, see also Figure 7):

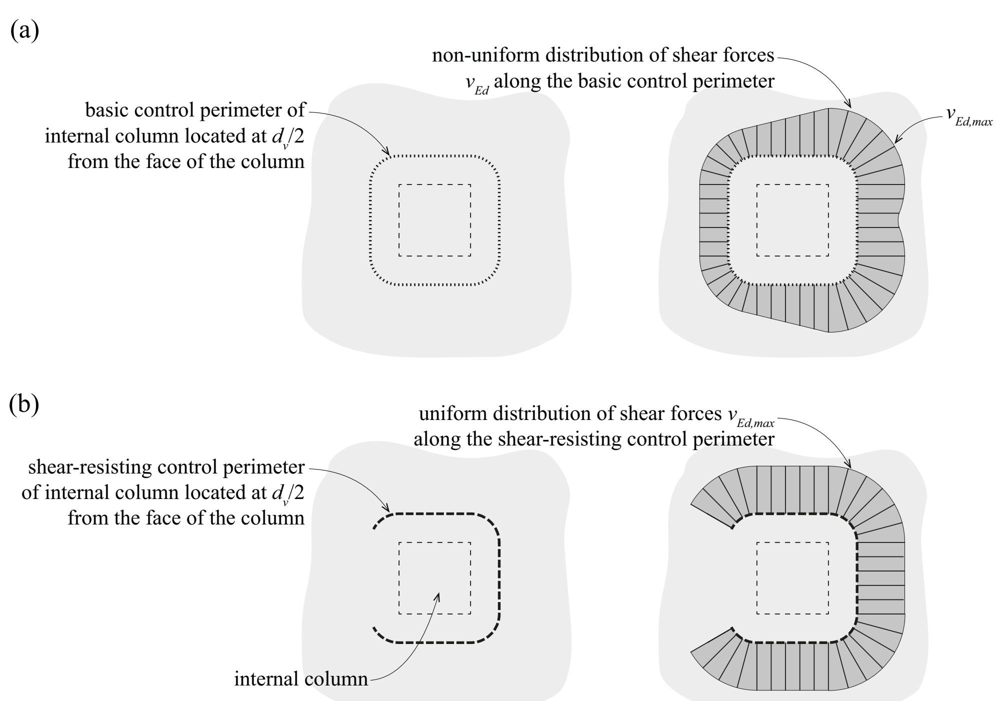

Figure 7. Eccentric punching: (a) non-uniform distribution of shear forces along the basic control perimeter; (b) idealized uniform distribution of shear forces along the reduced control perimeter.

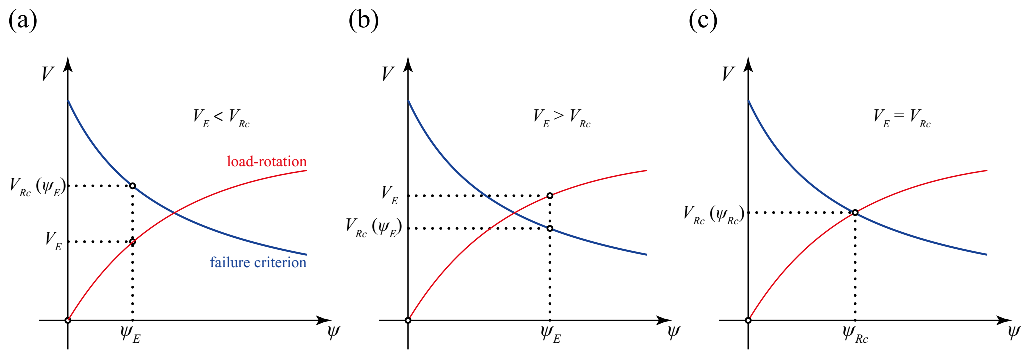

Figure 8. Application of Critical Shear Crack Theory for design and assessment: (a,b) procedure for design with calculation of punching shear resistance V_{Rc} corresponding to the rotation ψ_E associated to acting shear force V_E; (c) iteration required to calculate the punching resistance.

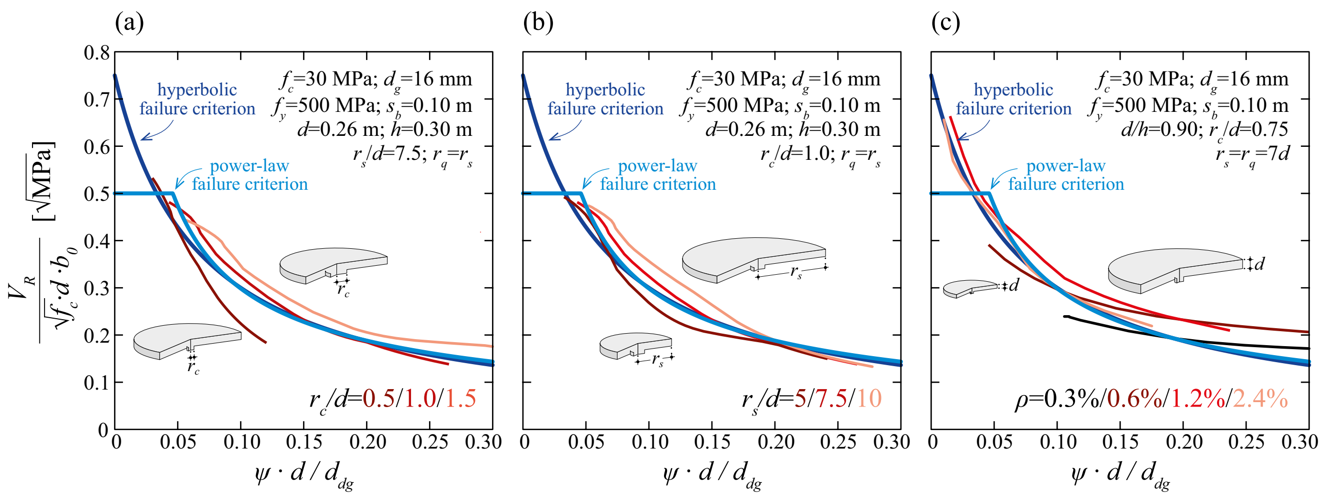

Figure 9. Comparison of original hyperbolic failure criterion (Eq. (2)) and failure criterion based on a power law (Eq. (9)) with the results from the refined mechanical model of CSCT by Simões et al. [48].

- a basic control perimeter, defined purely by geometric considerations, referring to the one located at $d_v/2$ (accounting for discontinuities, edges, opening, inserts and with straight segments limited to $3d_v$ in case of large columns or wall ends and corners).
- A reduced shear-resisting control perimeter which can be obtained by multiplying the basic control perimeter by a reduction factor k_e accounting for the concentrations of shear forces.

In this case, the concentration of shear forces is to be accounted for in the design of shear reinforcement by multiplying also A_{sw} (defined purely on the basis of geometry) by the coefficient ke in Eq. (5). Further details can be consulted elsewhere [65]. In a similar manner. the consideration of other effects such as elongated columns [66], prestressing [67, 68] or membrane forces [69, 70] can also be consistently accounted for.

3.4. Methodology for design and assessment of existing structures

Within the original formulation of the CSCT and its implementation in *fib* Model Code 2010 [25], it can be noted that the punching verification for design of a new structure is direct. This can be shown in Figures 8a,b, as it only has to be verified if the resistance is higher or equal than the demand for the rotation calculated by means of the load-rotation relationship. For an explicit calculation of the resistance, however, the two nonlinear curves shall be intersected (Figure 8c), which requires in general following an iterative procedure.

4. SIMPLIFICATIONS FOR DESIGN INTRODUCED IN FPREN 1992-1-1:2022

The implementation of the CSCT into the FprEN 1992-1-1:2022 [42] required several adaptions. The main one was to propose for design purposes a closed-form method for design as per EN 1992-1-1:2004 [8]. Such approach (where details can be consulted in [46]) allows for an analytical evaluation of the punching resistance (being thus direct both for design and assessment purposes) on the basis of a limited number of mechanical and geometrical parameters. For a detailed assessment of existing structures (defined in Annex I of FprEN1992-1-1:2022 [42]), the general method of the CSCT is however allowed, both in terms of failure criterion and general definition of the load-rotation relationship. This allows for a detailed evaluation of the load-rotation relationship accounting for the peculiarities of the structure.

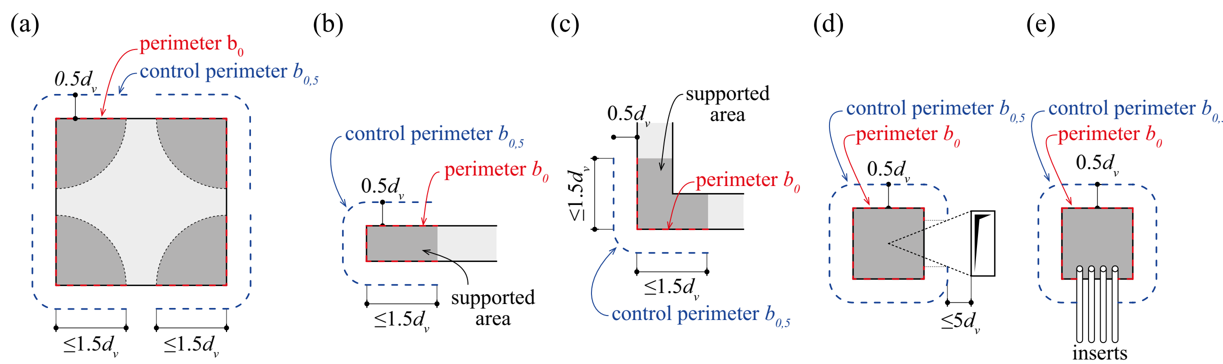

Figure 10. Definition of control perimeter $b_{0,5}$ at $d_v/2$ from the supported area and perimeter b0 at the face of the supported area

4.1. Members without shear reinforcement

In order to obtain a closed-form expression for punching, the failure criterion was slightly adapted [46, 72] from its hyperbolic form (Eq. (2)) to a power law with very similar results [48] (see Figure 9):

$$V_{R,c} = 0.55\, k_e\, b_{0,5}\, d_v \sqrt{f_c} \left(\frac{1}{25} \frac{d_{dg}}{\psi\, d}\right)^{\frac{3}{2}} \leq\ 0.50\, k_e\, b_{0,5}\, d_v \sqrt{f_c} \qquad (9)$$

where d_{dg} is the reference value of roughness of the critical shear crack and is computed as [46, 71]:

$$d_{gg} = d_{g0} + d_g \min\left(\left(\frac{60}{f_c}\right)^2, 1\right) \leq 40 \text{ mm} \qquad (10)$$

It can be noted that a value 0.50 is used in Eq. (9), instead of the original value of reference [46], for simplicity and yielding to almost identical results.

With respect to the load-rotation relationship, the one defined in Eq. (3) was adopted according to the presumed level of definition of the structure. It was however improved, as recent investigations suggest that the influence of the ratio a_p/d could be slightly modified to better approximate not only the theoretical response (e.g. integration of quadri-linear moment-curvature [28]) but also the punching shear resistance calculated with the refined model of the CSCT [48]. Accounting for such consideration, and assuming $m_s/m_R \approx V_E/V_{flex}$ and km≈1.2 [28], it results:

$$\psi = k_m \sqrt{8\, \frac{a_p}{d}}\, \frac{f_y}{E_s} \left(\frac{V_E}{V_{flex}}\right)^{\frac{3}{2}} \qquad (11)$$

where VE refers to the acting punching shear force and Vflex to the flexural capacity. The punching shear resistance can thus be directly determined by intersecting Eqs. (11) and (9), resulting into (see reference [74] for a complete derivation):

$$V_{R,c} = 0.16\, b_{0,5}\, d_v \sqrt{a\, \frac{d}{d_v}\, \frac{k_e\, d}{b_{0,5}}} \left(E_s\, \rho_l\, f_c \frac{d_{dg}}{\sqrt{d\, a_p}}\right)^{\frac{1}{3}} \leq 0.50\, k_e\, b_{0,5}\, d_v \sqrt{f_c} \qquad (12)$$

where $a = V_{flex}/m_R$. Eq. (12) can be written in a design format by (see reference [74] for derivation and associated considerations):

- using characteristic values of material strength and the partial safety factor associated to the required reliability index
- by considering additionally that the shear stress concentrations are not accounted for by reducing the control perimeter by a factor k_e (*fib* Model Code 2010 [25] approach) but rather by increasing the average acting shear stress calculated on the basic control perimeter by a coefficient β_e (Eurocode 2 [8] approach).
- for a safe simplified calculation, the parameter ap can be replaced by a value equal to 8*d*.
- replacing the term d by dv as a safe and simplified assumption (refer to Eq. (12) and see [74] for further details).

In that case, the design punching shear stress (to be compared with the acting shear stress $\tau_{Ed} = \beta_e\, V_{Ed}/(b_{0,5}\, d_v)$) becomes:

$$\tau_{Rd,c} = \frac{0.60}{\gamma_V}\, k_{pb}\, \left(100\, \rho_l\, f_{ck}\, \frac{d_{dg}}{d_V}\right)^{\frac{1}{3}} \leq \frac{0.50}{\gamma_V} \sqrt{f_{ck}} \qquad (13)$$

where d_{dg} is calculated according to Eq. (10) (with dg being replaced by the definition D_{lower}), ρ_l is the longitudinal flexural reinforcement ratio (with $\rho_l = \sqrt{\rho_{l,x}\ \rho_{l,y}}$, where subscripts *x,y* refer to two orthogonal directions). With respect to coefficient k_{pb} in Eq. (13), it accounts for the strength enhancement due to the shear field gradient in the control section and can be calculated as (see [74] for further details on the derivation):

$$1 \leq k_{pb} = 3.6 \sqrt{1 - \frac{b_0}{b_{0.5}}}\ \leq\ 2.5 \qquad (14)$$

where b_0 is the perimeter of the supporting area (perimeter at the column edge, see Figure 10). It should be noted that, for the sake of simplicity, kpb in Eq. (14) is expressed as a function of a geometrical rule using the two main control perimeters defined in FprEN 1992-1-1:2022 [42]: b_0 and $b_{0,5}$. This rule greatly simplifies notations for its practical use, hindering however the true physics of the phenomenon (a mechanical rule, as shown in [74], is replaced by a geometric one). This has to be kept in mind for the understanding of engineers of the design formulation.

It is interesting to note the physical meaning of the shear-gradient enhancement factor k_{pb}. This parameter describes the enhancement on the unitary shear resistance for a punching case with respect to the shear resistance of a beam or one-way slab. When the column (or in general the supporting area) is very large, k_{pb} tends to 1 and the punching shear resistance tends to the shear resistance of one-way slabs. Its value, otherwise, increases for decreasing column sizes, enhancing the unitary shear resistance. The upper limitation k_{pb} = 2.5 is addressed at very small columns.

With respect to the slenderness of the slab, it was stated before that a simplification was made on the load-rotation relationship as a safe bound ($a_p = 8d$) in Eq. (13). This is intended to increase the strains of the reinforcement and thus to reduce the unitary shear resistance. However, this consideration can be easily refined, by introducing a suitable strain effect, by replacing the parameter d_v by $\sqrt{d_v \frac{a_p}{8}}$, where:

$$a_p = \sqrt{a_{p,x}\ a_{p,y}} \geq d_v \tag{15}$$

where $a_{p,x}$ and $a_{p,y}$ are the distances between the column axis and the locations where the bending moments $m_{Ed,x}$ and $m_{Ed,y}$ are equal to zero.

4.2. Members with shear reinforcement

As it was done for members without shear reinforcement, several adaptions with respect to the CSCT general formulation were required to derive closed-form design expressions. These considerations are presented in the following for the three potential failure modes.

Failure within the shear-reinforced area

As previously introduced, the punching strength in case of failure within the shear reinforced region (VR,cs) is given by the sum of the contributions of concrete and shear reinforcement:

$$V_{R,cs}\ (\psi_E) = V_{R,c}\ (\psi_E) + k_e\, \sigma_{sw}\ (\psi_E)\ \Sigma A_{sw} \geq k_e\ f_{yw}\ \Sigma A_{sw} \tag{16}$$

where $V_{R,cs}$ (ψ_E) is the concrete contribution calculated with the failure criterion for the level of rotation ψ_E derived from the load-rotation relationship for the acting shear force; the term $\sigma_{sw}(\psi_E)$ is the stress in the shear reinforcement for the level of rotation ψ_E; the term f_{yw} is the yield strength of the shear reinforcement; k_e the coefficient accounting to the concentration of shear forces and ΣA_{sw} is the total area of shear reinforcement within $0.35{\cdot}d_v$ and d_v. In Eq. (16), the right-hand side of the inequality refers to the case where the concrete contribution vanishes ($V_{R,c}{\rightarrow}0$) and the stress in the shear reinforcement tends to the yielding strength ($\sigma_{sw}{\rightarrow}f_{yw}$) [64].

Eq. (16) was however considered not suitable for the punching design within the Eurocode 2 design philosophy since it is strain-based. To overcome that issue, Eq. (16) was simplified following an analytical derivation together with a number of simplifications (a detailed derivation is presented in [74]) :

- Replacing $V_{R,c}(\psi_E)$ by the corresponding value (Eq. (9) with ψ_E), but neglecting the upper limit;
- Introducing Eq. (11) (load-rotation) into Eq. (6) (activation of shear-reinforcement)
- Considering that $\eta_c = \frac{\tau_{Rd,c}}{\tau_{Ed}}$ and that $\tau_{Rd,c}$ is given by Eq. (13)
- Rounding the exponents and retaining only the most influential parameters for design

Following the above-mentioned considerations, Eq. (16) can be rewritten in a design format complying with the Eurocode 2 philosophy as (see [74] for further considerations):

$$\tau_{Rd,c} = \eta_c\ \tau_{Rd,c} + \eta_c\ f_{yw}\ \rho_w \geq f_{yw}\ \rho_{sw} \tag{17}$$

being:

$$\eta_c\ = \frac{\tau_{Rd,c}}{\tau_{Ed}} \tag{18}$$

$$\eta_s\ = \sqrt{15\,\frac{d_{dg}}{d_V}}\left(\frac{1}{\eta_c\, k_{pb}}\right)^{\frac{3}{2}} + \frac{d_v}{150\ \phi_w} \leq 0.80 \tag{19}$$

$$\rho_w\ = \frac{A_{sw}}{s_r\, s_t} \tag{20}$$

where A_{sw} is the area of one leg of shear reinforcement; sr is the radial spacing of shear reinforcement; s_t is the average tangential spacing of perimeters of shear reinforcement measured at control perimeter and f_{ywd} is the yield strength of the shear reinforcement. It should be noted that the factor $d_v/(150{\cdot}\varnothing_w)$ in Eq. (19) refers to the enhancement on the activation of the punching reinforcement due to bond, and thus that it can only be considered provided that the shear reinforcement consists of ribbed or indented bars.

Failure by crushing of concrete struts (maximum punching resistance)

According to the general frame of the CSCT [17], the maximum punching shear resistance of shear-reinforced slabs can be calculated by multiplying the concrete failure criterion by a factor ($\eta_{sys,sb}$), whose value accounts for the performance of the shear reinforcement system. Provided that the power-law failure criterion is multiplied by a factor, the resulting strength can also be obtained in a closed-form manner. For convenience, the strength will be expressed in this case on the basis of the one of a member without shear reinforcement:

$$V_{R,max} = \eta_{sys}\ V_{R,c} \tag{21}$$

where η_{sys} is the factor to enhance the punching resistance of slabs without shear reinforcement. In fact, factors $\eta_{sys,sb}$ and η_{sys} are related and account for the same effects. It shall be noted however that while the former is the multiplication factor to be applied in a strain-based approach (multiplication of the failure criterion), the latter is the multiplication factor to be applied to the punching resistance (their mathematical relationship is a function of the adopted failure criterion and load-rotation relationship).

In order to introduce in an explicit manner, the governing parameters ruling the value of η_{sys}, specific simulations were performed with the refined implementation of the CSCT [73]. It was found that the most influential parameters are (i) the type of punching reinforcement, (ii) the size of the column, (iii) the position of the first perimeter of punching reinforcement and (iv) the detailing of the anchorages (enclosure of the third or fourth layer of flexural reinforcement with the punching reinforcement units and the spacing of the subsequent perimeters). Other factors were also shown to have a certain impact (such as the yield strength and flexural reinforcement ratio), yet with a more limited impact for the daily design cases [73]. Based on this analysis [73], an analytical expression for the value of η_{sys} was formulated within FprEN1992-1-1:2022 [42] as:

$$\eta_{sys}\ = 1.15\,\frac{d_{sys}}{d_V} + 0.63\left(\frac{b_0}{d_V}\right)^{\frac{1}{4}} - 0.85\,\frac{s_0}{d_{sys}} \geq 1.0 \tag{22}$$

where d_{sys} represents the anchorage performance of the punching reinforcement system and its detailing and s_0 is the distance from the column face to the axis of the first perimeter of shear reinforcement. This expression fairly well approximates the results of the refined implementation of the CSCT and accounts for the effect of most detrimental parameters. For ease-of-use in designing new structures complying with the detailing rules of Section 12, constant values for the ratios d_{sys}/d_v and s_0/d_{sys} can be adopted depending on the type of shear reinforcement system, leading to the expressions included in Clause 8 of FprEN1992-1-1:2022 [42]:

$$\eta_{sys} = 0.70 + 0.63\left(\frac{b_0}{d_v}\right)^{\frac{1}{4}} \geq 1.0 \tag{23a}$$

$$\eta_{sys} = 0.50 + 0.63\left(\frac{b_0}{d_v}\right)^{\frac{1}{4}} \geq 1.0 \tag{23b}$$

whereas the more general expression of Eq. (22) is defined in Annex I for the assessment of existing structures.

Failure outside the shear-reinforced area

Following the general approach of the CSCT, the punching resistance outside the shear reinforced region should be calculated in accordance to Eq. (13), considering the reduced shear-resisting effective depth (function of the shear reinforcement system) and the outer control perimeter $b_{0,5,\text{out}}$ (located at $d_{v,out}/2$ from the outer perimeter of shear reinforcement with a length of the straight segments not exceeding $3d_{v,out}$).

5. COMPARISON OF FPREN 1992-1-1:2022 TO TESTS AND TO FIRST GENERATION OF EUROCODE 2

A systematic comparison of the formulation of FprEN 1992-1-1:2022 [42] against experimental tests was performed and published elsewhere [74]. No remarkable trend was observed, with a uniform level of safety and a relatively constant and low Coefficient of Variation (below or around 14% in all cases), improving the results by EN 1992-1-1:2004 [8]. The values obtained are amongst the lowest that can be found for any design code and comparable to those of the original theory.

6. CONSIDERATIONS FOR ASSESSMENT OF EXISTING STRUCTURES

The previous method was developed in order to provide designers with a simple tool for design, implying only a limited number of parameters and being sufficiently safe in the assumptions covering other (non-explicit) parameters. However, for assessment of existing structures, the different properties of the structure are usually known (in case drawings and documents are available) or can be assessed on-site (as the characteristic strength of concrete or the yield strength of the reinforcement). This allows one to perform more tailored analyses, with potential increases of the strength as the various load-carrying actions can be suitably evaluated, avoiding unnecessary strengthening or minimizing it.

To that aim, Annex I of the FprEN 1992-1-1:2022 [42] proposes a more general frame of verification, with an explicit definition of the failure criterion for punching based on the CSCT for members without shear reinforcement:

$$\frac{v_{R,c}}{b_{0,5}\, d_v} = 0.75\,\frac{\gamma_{def}^{\frac{2}{3}}}{\gamma_v^2}\,\frac{\sqrt{f_{ck}}}{1+15\,\dfrac{\gamma_{def}\,\psi\, d_v}{d_{dg}}} \tag{24}$$

This failure criterion is equivalent to the one defined by the CSCT (refer to Eq. (2)), but accounting for partial safety factors (γ_{def} and γ_v) to comply with the required level of reliability [75] (similar considerations as for the general CSCT approach can be assumed for other failure modes).

The rotation at failure can be estimated by intersection of the failure criterion with the load-rotation -relationship at the slab-column connection. This latter can be calculated accounting for the different geometrical and mechanical conditions. For instance, an analysis based on nonlinear finite elements is a suitable strategy for this purpose [e.g. 58,59], although simpler approaches might be sufficient.

7. EXAMPLE OF APPLICATION

An example is presented in the following referring to the assessment of the punching resistance of an existing structure. To that aim, the geometry and reinforcement layout are considered as known data. The assessment of the resistance is performed first by using the closed-form approach for design provided in Clause 8.4 (Eq. (13)), whose value is later refined by means of the strain-based approach and consideration of membrane action according to Annex I. The example is inspired on a real structure built in Lausanne, Switzerland during the 1990s, serving as a hall for maintenance of vehicles. The most relevant properties for the assessment of the slab without shear reinforcement are listed below:

- ***Geometry***: the geometry of the slab considered in the design example is shown in Figure 11.
  - Slab's overall depth: h = 0.32 m
  - Spans: L_x = 7.80 m; L_y = 8.00 m
  - Cover: 20 mm
  - Effective depth: d = 0.28 m
  - Shear-resisting effective depth: d_v = 0.28 m (0.00 m column penetration)
  - Columns: square 0.50 x 0.50 m
- ***Materials***
  - Concrete: f_{ck} = 42.8 MPa (measured *in-situ* and calculated according to Annex I of FprEN1992-1-1:2022 [42] on the basis of $f_{ck,is}$ determined with EN 13791 [76]); D_{max} = 32 mm
  - Flexural reinforcement: B500; B ductility class; $f_{yd} = f_{ywd}$ = 435 MPa
- ***Top flexural reinforcement***: Ø18@0.10 m in both x- and y- directions

  Partial safety factors: γ_V = 1.4; γ_{def}=1.33
- Acting shear force: V_{Ed} = 1.167 MN

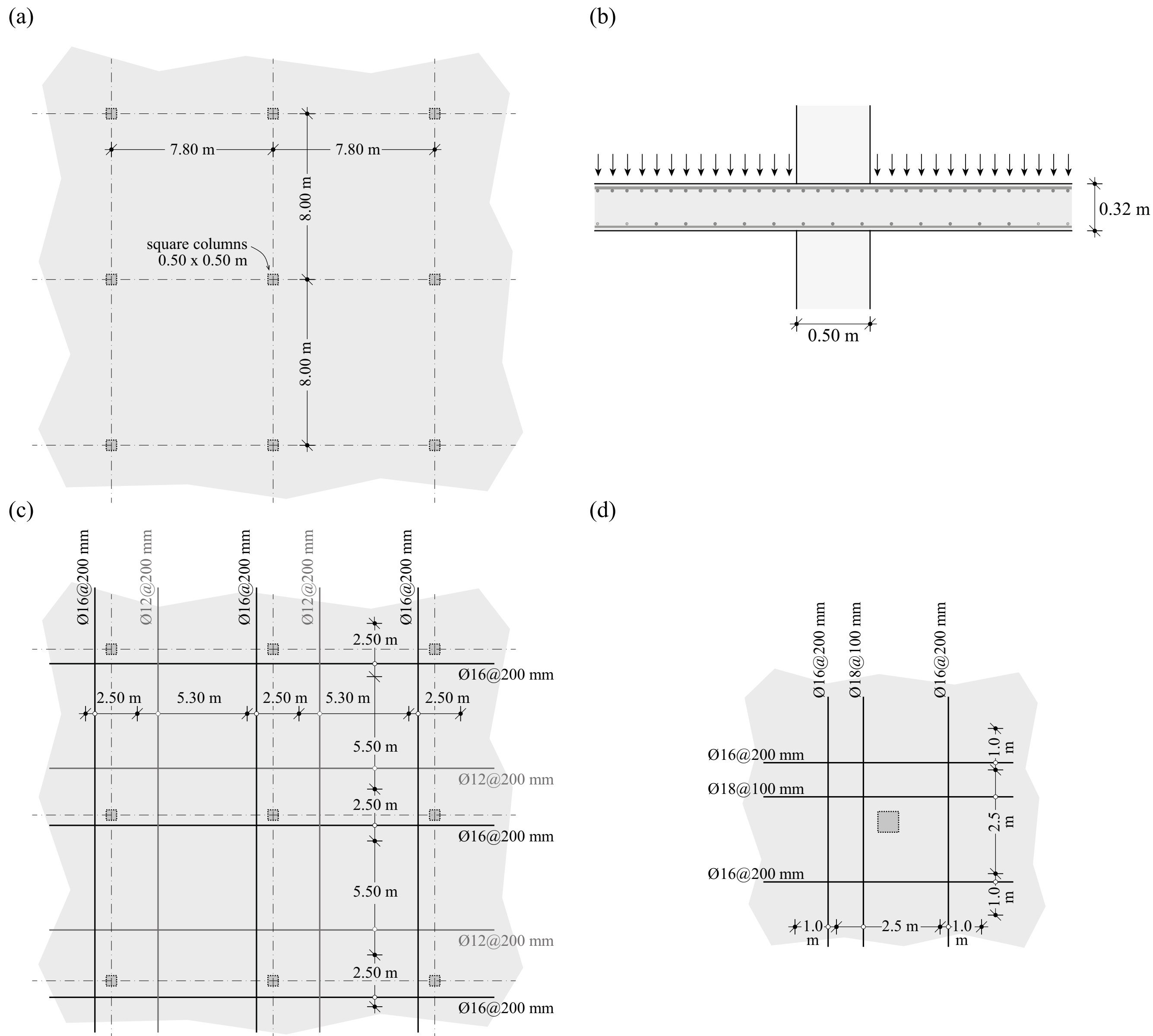

Figure 11. Example of assessment of existing structure: (a) geometry in plan; (b) cross section of slab-column connection; (c) bottom flexural reinforcement ratio; and (d) top flexural reinforcement ratio.

Punching resistance according to Section 8.4 of FprEN 1992-1-1:2022 [42]

The control perimeters b0 and b0,5 are given by:

$$b_0 = 4\ \ 0.50 = 2.0 \text{ m}$$
$$b_{0,5} = b_0 + 2\ \ \pi\ \ d_v/2 = 2.0 + \pi\ \ 0.28 = 2.88 \text{ m}$$

The value of the parameter β_e accounting for concentrations of the shear forces due to moment transfer between the slab and the column can be assumed equal to β_e =1.15 according to clause 8.4.2(6) (it could also be calculated following a refined methodology). The acting shear stress τ_{Ed} is thus given by:

$$\tau_{Ed} = \frac{\beta_e\ V_{ed}}{b_{0,5}\ \ b_v} = \frac{1.15 \cdot 1.167}{2.88 \cdot 0.28} = 1.66 \text{ MPa}$$

With respect to the shear stress resistance without shear reinforcement, it is given by:

$$d_{dg} = \ 0.016 + 0.032 = 0.048 \le 0.040 \Rightarrow d_{dg} = 0.040 \text{ m}$$

$$k_{pb} = 3.6\sqrt{1 - \frac{b_0}{b_{0,5}}} = 3.6\ \ 1 - \frac{2.00}{2.88} = 1.99 \quad \text{with } 1 \le k_{pb} = 1.99 \le 2.5$$

$$\tau_{Rd,c} = \frac{0.60}{1.40}\ \ 1.99\left(100 \cdot 0.0091 \cdot 42.8\ \ \frac{0.040}{0.28}\right)^{\frac{1}{3}} \le \frac{0.50}{1.40}\sqrt{42.8}$$

$$\Rightarrow \tau_{Rd,c} = 1.51 \le 2.34 \quad \Rightarrow \quad \tau_{Rd,c} = 1.51 \text{ MPa}$$

As $\tau_{Ed} > \tau_{Rd,c}$, the punching shear resistance without shear reinforcement is insufficient. As $a_p = \sqrt{0.22\ \ 8.0\ \ 0.22\ \ 7.80} = 1.74$ m $< 8d_v = 2.24$ m, the punching shear resistance can still be increased with clause 8.4.3(2) adopting $a_{pd} = \sqrt{1.74 \dfrac{0.28}{8}} = 0.247\ m$:

$$\tau_{Rd,c} = \frac{0.60}{1.40}\ 1.99 \left(100 \cdot 0.0091 \cdot 42.8\ \frac{0.040}{0.28}\right)^{\frac{1}{3}} \leq \frac{0.50}{1.40}\sqrt{42.8}$$

$$\Rightarrow \tau_{Rd,c} = 1.51 \leq 2.34 \quad \Rightarrow \quad \tau_{Rd,c} = \underline{1.57\ \text{MPa}}$$

As $\tau_{Ed} > \tau_{Rd,c}$, the punching shear resistance without shear reinforcement is again insufficient following the formulae proposed in Section 8.4 which is tailored for the design of new structures. The Annex I, for existing structures, can be used.

Punching resistance according to Annex I of FprEN 1992-1-1:2022 [42]

Annex I of FprEN 1992-1-1:2022 allows calculating the punching resistance by intersection of the load-rotation relationship and the failure criterion of Eq. (24). This procedure is shown in Figure 12 for the example presented in this section.

In this case, the most accurate load-rotation relationship (red line) is calculated considering a layered sectional model calculated with finite elements accounting for the non-linear behaviour of the concrete and reinforcement (considering tension-stiffening effects and reinforcement yielding). This approach has been assessed by comparing the calculated load-rotation relationship with the experimental values of several benchmark tests. To that aim, the methodology explained in [59] is followed, where the governing rotation is measured at a distance $2d_v$ from the control perimeter. The design shear stress τ_{Ed} =1.59 MPa is calculated in accordance to clause 8.4.2(6) with the coefficient accounting for the concentration of shear forces (β_e) computed with the refined approach (Table 8.3 of FprEN 1992-1-1:2022 [42]; β_e=1+1.1·e_b/b_b=1.10).

The design punching shear resistance obtained following this procedure is equal to 1.69 MPa, being approximately 10% larger than the one calculated with the closed-form formulae of Section 8.4 (1.57 MPa). Such value allows verifying that the punching resistance is sufficient. In terms of the compliance factor for punching resistance ($\tau_{Rd,c}/\tau_{Ed}$), it increases from 0.95 to 1.06 using Annex I from FprEN 1992-1-1:2022. This allows justifying the structural safety related to punching failures, avoiding expensive (or unnecessary) strengthening measures. The increase on the resistance is in this case mainly associated to the non-linear response of the slab (which accounts for the slab continuity and membrane action). Such effects lead to a stiffer response when compared to the load-rotation relationship obtained with the parabola of Eq. (11) (represented by the dashed black line in Figure 12). As a consequence of the stiffer response, narrower crack widths can be expected and consequently a higher punching resistance.

It shall be noted that Annex I of FprEN 1992-1-1:2022 [42] also allows accounting for the favourable effect of compressive membrane action around internal columns (in absence of large openings or inserts in the vicinity of the column) based on the closed-form expressions for the punching resistance. This is performed by multiplying the factor k_{pb} by an enhancement factor η_{pm} (Clause I.8.5.1). Applying such clause to the present example leads to a punching resistance equal to 1.75 MPa. This result is comparable to the one obtained based on the nonlinear analysis. Such good agreement between the nonlinear analysis and the closed-form expression with membrane action enhancement is generally found for typical cases if internal columns (comprising regular geometries and usual reinforcement arrangements). For unusual geometries or reinforcement layouts, as well as for corner or edge columns, the nonlinear analysis of the flat slab allows better considering the actual response of the system and leads generally to higher estimates of the resistance.

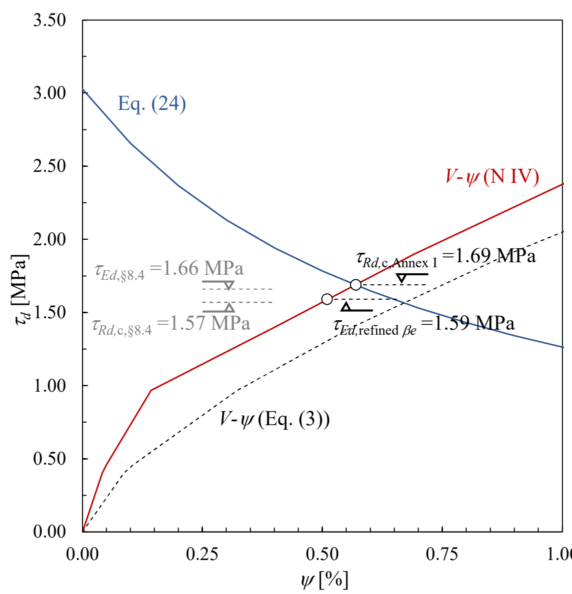

Figure 12. Assessment of existing slab-column connection according to Annex I of FprEN 1992-1-1:2022 [42].

8. CONCLUSIONS

The new provisions for Eurocode 2 (FprEN 1992-1-1:2022 [42]) with respect to punching verification have underwent some major changes. The most significant aspect is that the code, previously based on an empirical formula, has now been based on a mechanical model. This allows for:

- Enhanced consistency of the provisions, with consideration of the different phenomena (such as size and strain effects) in a sound manner
- Allowing for a transparent understanding of the design expressions and the role of the various geometrical and mechanical parameters implied
- Lead to simple formulations for design, but providing a general frame for a more accurate assessment of existing structures

The Critical Shear Crack Theory (CSCT) was selected as the theory to ground the punching shear provisions, but its implementation as performed in *fib* Model Code 2010 was however considered inconvenient for the FprEN 1992-1-1:2022 [42]. Thus, the theory was implemented in an alternative manner considering:

An explicit closed-form formulation for design and simple assessment based on a limited number of physical and mechanical parameters. This required some adaptions from the classical formulation, comprising a new definition of the failure criterion and introducing a number of simplifications for ease-of-use.

A general and flexible framework to assess in a detailed manner the punching resistance when the geometrical and mechanical properties of a structure are known in detail. This approach implies intersecting the failure criterion of the CSCT with a suitable load-rotation relationship. Such methodology is typically convenient for assessment of critical existing structures and is provisioned into the Annex I of FprEN 1992-1-1:2022 [42], addressed at existing structures

The proposed approach is shown to lead to consistent results when compared to available test results, and also to be simple to use for practical purposes.

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References

[1] Fernández Ruiz M., Mirzaei Y., Muttoni A., *Post-Punching Behavior of Flat Slabs*, ACI Structural Journal, V. 110, USA, 2013, pp. 801-812. https://doi.org/10.14359/51685833

[2] Silfwerbrand J., Hassanzadeh G., Editors., *International Workshop on punching shear capacity of RC slabs*, Royal Institute of Technology, Stockholm, Sweden, 2000, 527 p.

[3] *fib*, *Punching of structural concrete slabs*, *fib* Bulletin 12, Lausanne, Switzerland, 2001, 307 p. https://doi.org/10.35789/fib.BULL.0012

[4] Polak M. A., Editor, *Punching Shear in Reinforced Concrete Slabs*, SP-232, American Concrete Institute, Farmington Hills, MI, 2005, 302 p. https://doi.org/10.14359/14960

[5] *fib*, *Shear and punching shear in RC and FRC elements*, *fib* Bulletin 57, Lausanne, Switzerland, 2010, 268 p. https://doi.org/10.35789/fib.BULL.0057

[6] *fib*, Punching shear of structural concrete slabs, *fib* Bulletin 81, Lausanne, Switzerland, 2017, 378 p.

[7] Comite Euro-International du béton, CEB-FIP Model Code 1990: Design Code, Thomas Telford Ltd., London, England, 1993, 460 p.

[8] CEN, *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings*, EN 1992-1-1, Brussels, Belgium, 2004, 225 pp.

[9] ACI, *Building Code Requirements for Reinforced Concrete* (ACI 318-14), Farmington Hills, USA, 2014, 524 p.

[10] ACI, *Building Code Requirements for Reinforced Concrete* (ACI 318-19), Farmington Hills, USA, 2019, 624 p.

[11] Zsutty, T.C., *Beam Shear Strength Prediction by Analysis of Existing Data*, ACI Journal Proceedings, V. 65, N. 11, 1968, pp. 943-951

[12] Kinnunen S., Nylander H., *Punching of Concrete Slabs Without Shear Reinforcement*, Transactions of the Royal Institute of Technology, N. 158, 1960, 112 p.

[13] Muttoni, A., *Punching shear – Draft code proposal, SIA 162*, Working Group 5, Swiss Society of Engineers and Architects, Zürich, 1985, 15 p.

[14] SIA, *Code 162 for Concrete Structures*, Swiss Society of Engineers and Architects, Zürich, 1993, 86 p.

[15] Muttoni, A., and Schwartz, J., *Behaviour of Beams and Punching in Slabs without Shear Reinforcement*, IABSE Colloquium, V. 62, Zürich, Switzerland, 1991, pp. 703-708

[16] Muttoni A., *Schubfestigkeit und Durchstanzen von Platten ohne Querkraftbewehrung, Beton- und Stahlbetonbau* (In German), Beton-und Stahlbetonbau, V. 98, 2003, pp. 74-84 https://doi.org/10.1002/best.200300400

[17] Fernández Ruiz M., Muttoni A., *Applications of the critical shear crack theory to punching of R/C slabs with transverse reinforcement*, ACI Structural Journal, V. 106, N. 4, 2009, pp. 485-494 https://doi.org/10.14359/56614

[18] Muttoni A., Fernández Ruiz M., Guandalini S., *Poinçonnement des ponts-dalles*, Documentation SIA, 4. FBH/ASTRA Studientagung 'Neues aus der Brückenforschung', 2007, pp. 85-94

[19] Clément T., *Influence de la précontrainte sur la résistance au poinçonnement de dalles en béton armé* (In French), Doctoral Thesis, EPFL, Lausanne, Switzerland, 2012, N. 5516, 222 p.

[20] Simões J. T., *The mechanics of punching in reinforced concrete slabs and footings without shear reinforcement*, Doctoral thesis, EPFL, Lausanne, Switzerland, 2018, N. 8387, 223 p.

[21] Maya Duque L. F., Fernández Ruiz M., Muttoni A., Foster S. J., *Punching shear strength of steel fibre reinforced concrete slabs*, Engineering Structures, V. 40, 2012, pp. 93-94. https://doi.org/10.1016/j.engstruct.2012.02.009

[22] Fernández Ruiz M., Muttoni A., Kunz J., *Strengthening of flat slabs against punching shear using post-installed shear reinforcement*, ACI Structural Journal, V. 107 N° 4, USA, 2010, pp. 434-442. https://doi.org/10.14359/51663816

[23] Faria D. M. V., Einpaul J., Ramos A.P., Fernández Ruiz M., Muttoni A., *On the efficiency of flat slabs strengthening against punching using externally bonded fibre reinforced polymers*, Construction and Building Materials, V. 73, 2014, pp. 366-377. http://dx.doi.org/10.1016/j.conbuildmat.2014.09.084

[24] Muttoni A., Fernández Ruiz M., *The levels-of-approximation approach in MC 2010: application to punching shear provisions*, Structural Concrete, V. 13, N. 1, 2012, pp. 32-41. https://doi.org/10.1002/suco.201100032

[25] Fédération internationale du béton, *fib* Model Code for Concrete Structures 2010, Ernst & Sohn, Germany, 2013, 434 p.

[26] Muttoni A., Fernández Ruiz M., Bentz E., Foster S., Sigrist V., *Background to fib Model Code 2010 shear provisions – part II: punching shear*, Structural Concrete, V. 14, N. 3, 2013, pp. 204-214 https://doi.org/10.1002/suco.201200064

[27] SIA, *Code 262 for Concrete Structures*, Zürich, Switzerland: Swiss Society of Engineers and Architects, 2013, 102 p.

[28] Muttoni A., *Punching shear strength of reinforced concrete slabs without transverse reinforcement*, ACI Structural Journal, V. 105, N. 4, 2008, pp. 440-450. https://doi.org/ 10.14359/19858

[29] Hegger J., Ricker M., Sherif A. G., *Punching Strength of Reinforced Concrete Footings*, ACI Structural Journal, V. 106, N. 5, 2009, pp. 706-716 https://doi.org/10.14359/51663111

[30] Vollum R.L., Abdel-Fattah T., Eder M., Elghazouli A.Y., *Design of ACI-type punching shear reinforcement to Eurocode 2*, Magazine of Concrete Research, V. 62, N. 1, 2010, pp. 3-16 https://doi.org/10.1680/macr.2008.62.1.3

[31] Siburg C., Haüsler F., Hegger J., *Flat slab punching design according to German annex of Eurocode 2*, Bauingenieur, V. 87, 2012, pp. 216-225

[32] Siburg C., Ricker M., Hegger J., *Punching shear design of footings: critical review of different code provisions*, Structural Concrete, V. 15, 2014, pp. 497-508 https://doi.org/10.1002/suco.201300092

[33] Ricker M., Siburg C., *Punching shear strength of flat slabs – critical review of Eurocode 2 and fib Model Code 2010 design provisions*, Structural Concrete, V. 17, 2016, pp. 457-468 https://doi.org/10.1002/suco.201500106

[34] Einpaul, J., Bujnak, J., Fernández Ruiz, M., Muttoni, A., *Study on influence of column size and slab slenderness on punching strength*, ACI Structural Journal, V. 113, N. 1, 2016, pp. 135-145 https://doi.org/10.14359/51687945

[35] Einpaul J., Brantschen F., Fernández Ruiz M., Muttoni A., *Performance of Punching Shear Reinforcement under Gravity Loading: Influence of Type and Detailing*, ACI Structural Journal, V. 113, N. 4, 2016, pp. 827-838 https://doi.org/10.14359/51688630

[36] Kueres D., Siburg C., Herbrand M., Classen M., Hegger J., *Uniform Design Method for punching shear in flat slabs and column bases*, Engineering Structures, V. 136, 2017, pp. 149-164

https://doi.org/10.1016/j.engstruct.2016.12.064

[37] Fernández Ruiz M., Muttoni A., *Size effect in shear and punching shear failures: differences between statically determinate members and redundant structures*, Structural Concrete, V. 19, 2018, pp. 65-75, https://doi.org/10.1002/suco.201700059

[38] ECP, *Eurocode Commentary* (rev A 31-03-2017), European Concrete Platform, Brussels, Belgium, 2008, 220 p.

[39] HILTI, *Fastening Technology Manual, Post-installed punching reinforcement Hilti HZA-P*, Hilti, 2019, 33 p.

[40] CEN, *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings*, Corrigendum, EN 1992-1-1:2004+AC:2010, Brussels, Belgium, 2010, 23 pp.

[41] CEN, *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings*, EN 1992-1-1:2004+A1:2014, Brussels, Belgium, 2014, 6 pp.

[42] CEN/TC 250, *Eurocode 2: Design of Concrete Structures—Part 1-1: General Rules - Rules for buildings, bridges and civil engineering structures*, FprEN 1992-1-1:2022, Brussels, Belgium, 2022-05, 408 pp (this document is available through the National members at CEN TC250/SC2).

[43] Walkner R., Kritische Analyse des Durchstanznachweises nach EC2 und Verbesserung des Bemessungsansatzes (In German), Doctoral Thesis, Innsbruck University, 2014, 458 p.

[44] Muttoni A., Fernández Ruiz M., Simões J.T., *The theoretical principles of the Critical Shear Crack Theory for punching shear failures and derivation of consistent closed-form design expression*, Structural Concrete, V. 19, 2018, pp. 174-190, https://doi.org/10.1002/suco.201700088

[45] Fernández Ruiz M., Sagaseta Albajar J., Muttoni A., *La teoría de la fisura crítica como base teórica para el diseño de losas frente a punzonamiento en el nuevo Código Modelo 2010*, Hormigón Y Acero, V.63, N. 263, pp. 49-63, 2012.

[46] Muttoni A., Fernández Ruiz M., Simões J.T., *The theoretical principles of the Critical Shear Crack Theory for punching shear failures and derivation of consistent closed-form design expression*, Structural Concrete, V. 19, 2018, pp. 174-190, https://doi.org/10.1002/suco.201700088

[47] Muttoni A., Fernández Ruiz M., *Shear in slabs and beams: should they be treated in the same way?*, *fib* bulletin 57, Lausanne, Switzerland, 2010, pp. 105-128 https://doi.org/10.35789/fib.bull.0057.ch07

[48] Simões J.T., Fernández Ruiz M., Muttoni A., *Validation of the Critical Shear Crack Theory for punching of slabs without transverse reinforcement by means of a refined mechanical model*, Structural Concrete, V. 19, 2018, pp. 191–216, https://doi.org/10.1002/suco.201700280

[49] Muttoni A, Fernández Ruiz M., Simões J. T., *Recent improvements of the Critical Shear Crack Theory for punching shear design and its simplification for code provisions*, Proceedings of 5th *fib* Symposium, Melbourne, 2018, 14 p.

[50] Einpaul J., Fernández Ruiz M., Muttoni A., *Measurements of internal cracking in punching tests slabs without shear reinforcement*, Magazine of Concrete Research, V. 70, N. 15, 2018, pp. 798-810 https://doi.org/10.1680/jmacr.16.00099

[51] Vecchio F.J., Collins M.P., *The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear*, ACI Journal Proceedings, V. 83, N. 2, 1986, pp. 219-231 https://doi.org/10.14359/10416

[52] Guidotti R., *Poinçonnement des planchers-dalles avec colonnes superposées fortement sollicitées* (In French), Doctoral Thesis, EPFL, Lausanne, Switzerland, 2010, N. 4812, 189 p.

[53] Walraven J.C., *Fundamental Analysis of Aggregate Interlock*, Journal of Structural Engineering, ASCE, V. 107, N. 11, 1981, pp. 2245-2270

[54] Tirassa M., Fernández Ruiz M., Muttoni A., *An interlocking approach for the rebar-to-concrete contact in bond*, Magazine of Concrete Research, V. 73, N. 8, 2021, pp. 379-393 https://doi.org/10.1680/jmacr.20.00209

[55] Fernández Ruiz M., *The influence of the kinematics of rough surface engagement on the transfer of forces in cracked concrete*, Engineering Structures, V. 231, N. 15, 2021, 17 p. https://doi.org/10.1016/j.engstruct.2020.111650

[56] Belletti B., Walraven J.C., Trapani F., *Evaluation of compressive membrane action effects on punching shear resistance of reinforced concrete slabs*, Engineering Structures, V. 95, 2015a, pp. 25-39 https://doi.org/10.1016/J.ENGSTRUCT.2015.03.043

[57] Belletti B., Pimentel M., Scolari M., Walraven J.C., *Safety assessment of punching shear failure according to the level of approximation approach*, Structural Concrete, V. 16, N. 3, 2015b, pp. 366-380 https://doi.org/10.1002/suco.201500015

[58] Setiawan A., Vollum R.L., Macorini L., Izzuddin B.A., *Numerical modelling of punching shear failure of reinforced concrete flat slabs with shear reinforcement*, Magazine of Concrete Research, V. 73, N. 23, 2021, pp. 1205-1224 https://doi.org/10.1680/jmacr.19.00562

[59] Setiawan A., Viúla Faria D.M., Hernández Fraile D., Fernández Ruiz M., Muttoni A., *On the behaviour of exterior slab panels of flat slab buildings*, *fib* Symposium, Lisbon, June 2021, 12 p.

[60] Beutel R., Hegger J., *The effect of anchorage on the effectiveness of the shear reinforcement in the punching zone*, Cement and Concrete Composites, V. 24, 2002, pp. 539-549 https://doi.org/10.1016/S0958-9465(01)00070-1

[61] Lips S., Fernández Ruiz M., Muttoni A., *Experimental Investigation on Punching Strength and Deformation Capacity of Shear-Reinforced Slabs*, ACI Structural Journal, V. 109, 2012, pp. 889-900 https://doi.org/10.14359/51684132

[62] Simões J.T., Bujnak J., Fernández Ruiz M., Muttoni A., *Punching shear on compact footings with uniform soil pressure*, Structural Concrete, V. 17, N. 4, 2016, pp. 603-617 https://doi.org/10.1002/suco.201500175

[63] Simões J.T., Viúla Faria D.M., Fernández Ruiz M., Muttoni A., *Strength of reinforced concrete footings without transverse reinforcement according to limit analysis*, Engineering Structures, V. 112, 2016, pp. 146-161 https://doi.org/10.1016/j.engstruct.2016.01.010

[64] Brantschen F., Viúla Faria D.M., Fernández Ruiz M., Muttoni A., *Bond Behaviour of Straight, Hooked, U-Shaped and Headed Bars in Cracked Concrete*, Structural Concrete, V. 17, N. 5, 2016, pp. 799-810 https://doi.org/10.1002/suco.201500199

[65] Sagaseta J., Muttoni A., Fernández Ruiz M., Tassinari L., *Non-axis-symmetrical punching shear around internal columns of RC slabs without transverse reinforcement*, Magazine of Concrete Research, V. 63, N. 6, 2011, pp. 441-457 https://doi.org/10.1680/macr.10.00098

[66] Sagaseta J., Tassinari L., Fernández Ruiz M., Muttoni A., *Punching of flat slabs supported on rectangular columns*, Engineering Structures, V. 77, 2014, pp. 17-33 https://doi.org/10.1016/j.engstruct.2014.07.007

[67] Clément T., Pinho Ramos A., Fernández Ruiz M., Muttoni A., *Influence of prestressing on the punching strength of post-tensionned slabs*, Engineering Structures, V. 72, 2014, pp. 59-69 https://doi.org/10.1016/j.engstruct.2014.04.034

[68] Clément T., Pinho Ramos A., Fernández Ruiz M., Muttoni A., *Design for punching of prestressed concrete slabs*, Structural Concrete, V. 14, N. 2, 2013, pp. 157-167 https://doi.org/10.1002/suco.201200028

[69] Einpaul J., Ospina C. E., Fernández Ruiz M., Muttoni A., *Punching Shear Capacity of Continuous Slabs*, ACI Structural Journal, V. 113, N. 4, 2016, pp. 861-872 https://doi.org/10.14359/51688758

[70] Einpaul J., Fernández Ruiz M., Muttoni A., *Influence of moment redistribution and compressive membrane action on punching strength of flat slabs*, Engineering Structures, V. 86, 2015, pp. 43-57 https://doi.org/10.1016/j.engstruct.2014.12.032

[71] Cavagnis F. Fernández Ruiz M., Muttoni A., *A mechanical model for failures in shear of members without transverse reinforcement based on development of a critical shear crack*, Engineering Structures, V. 157, 2018, pp. 300-315 https://doi.org/10.1016/j.engstruct.2017.12.004

[72] Muttoni A., Fernández Ruiz M., *The Critical Shear Crack Theory for punching design: from a mechanical model to closed-form design expressions*, ACI/*fib* International Punching Shear Symposium, Bulletin *fib* 81, 2017, pp. 237-252

[73] Hernández Fraile D., Simões J. T., Fernández Ruiz M., Muttoni A., *A mechanical approach for the maximum punching resistance of shear-reinforced slab-column connections*, *fib* Symposium, Lisbon, June 2021, 12 p.

[74] Muttoni A., Fernández Ruiz M., Simões J.T., Hernández Fraile D., Hegger J., Siburg C., Kueres D., *Background document to section 8.4 – Punching (FprEN 1992-1-1:2022)*, Report EPFL 17-01-R5, December 2022 (in press)

[75] Muttoni A., Yu Q., *Partial safety factor for strain-based formulae*, presentation to *fib* TG3.1, Oslo, June 2022, 12 p.

[76] CEN, EN 13791: Assessment of in-situ compressive strength in structures and precast concrete components, Brussels, Belgium, 2019, 41 p.

Notation

A_{sw}	area of a unit of shear reinforcement
$C_{Rd,c}$	NDP from NP EN 1992-1-1:2004 [8] with proposed value equal to 0.18/γC
E_s	modulus of elasticity of the flexural reinforcement
E_{sw}	modulus of elasticity of the shear reinforcement
L	spans (indices referring to directions)
V_E	acting punching shear force
V_{Ed}	design acting punching shear force
V_{flex}	flexural capacity
$V_{R,c}$	punching shear resistance for members without shear reinforcement
$V_{R,cs}$	punching shear resistance for failures within the shear-reinforced area(concrete contribution)
$V_{R,c,cs}$	concrete contribution for failures within the shear-reinforced area
$V_{R,s,cs}$	steel contribution for failures within the shear-reinforced area~
$V_{R,max}$	punching resistance associated to the crushing of the concrete struts
$V_{R,out}$	punching resistance of failures outside of the shear-reinforced area
a_p	distance between the axis of the column and the line of zero radial moment
b_s	support strip width
b_0	perimeter of the support region (perimeter at the column edge minimised for re-entrant corners and columns near to the edge, see Figure 10)
$b_{0,5}$	control perimeter at a distance of $d_v/2$ from the column face (round corners in case of square or rectangular columns)
$b_{0,5,out}$	outer control perimeter
d	effective depth
d_{dg}	reference value of roughness of the critical shear crack
d_{g0}	reference aggregate size (d_{g0} = 16 mm for normal weight concrete)
d_v	shear resisting effective depth
$d_{v,out}$	shear-resisting effective depth of the outer perimeter of reinforcement
d_{sys}	the anchorage performance of the punching reinforcement system and its detailing
f_b	average bond stress
f_c	cylinders concrete compressive strength
f_y	yield strength of the flexural reinforcement
f_{yw}	yield strength of the shear reinforcement
f_{ywd}	yield strength of the shear reinforcement
f_{ck}	cylinders characteristic concrete compressive strength
$f_{ck,is}$	cylinders characteristic concrete compressive strength measured in-situ
k	factor accounting for the size effect in NP EN 1992-1-1:2004 [8]
k_m	factor depending on the level of the refinement of the approach used to estimate the acting bending moment in the support width (typically 1.5 for simple analyses and 1.2 when some parameters are known more in detail)
k_{pb}	punching strength enhancement factor due to the shear field gradient in the control section in FprEN 1992-1-1:2022 [42]
k_e	reduction factor to be multiplied to the basic control perimeter to account for the concentrations of shear forces
k_1	NDP from NP EN 1992-1-1:2004 [8] with proposed value equal to 0.1
m_s	acting bending moment in the support strip width
m_R	average moment capacity in the support width
$v_{Rd,c}$	punching shear resistance in the basic control section [MPa] according to NP EN 1992-1-1:2004 [8]
v_{min}	NDP from NP EN 1992-1-1:2004 [8] with proposed value equal to 0.035 k3/2 fck1/2
s_r	radial spacing of shear reinforcement between the first and second unit
s_t	average tangential spacing of perimeters of shear reinforcement measured at control perimeter
s_0	distance from the column face to the axis of the first perimeter of shear reinforcement
ΣA_{sw}	total area of the activated shear reinforcement in the punching cone
β_e	coefficient to increase the average acting shear stress on the basic control perimeter to account for the concentrations of shear stresses in FprEN 1992-1-1:2022 [42]
γ_C	partial safety factor in NP EN 1992-1-1:2004 [8]
γ_{def}	partial safety factor for the rotation in the strain-based approach in Annex I of FprEN 1992-1-1:2022 [42]
γ_V	partial safety factor for shear design in FprEN 1992-1-1:2022 [42]
φ_w	diameter of a shear reinforcement unit
η_c	factor accounting for the reduction of the concrete contribution to the punching resistance with increasing rotation
η_{pm}	enhancement factor accounting for the favourable effect of compressive membrane action in Annex I of FprEN 1992-1-1:2022 [42]
η_s	factor accounting for the increase of the shear reinforcement contribution to the punching resistance with increasing rotation
η_{sys}	enhancement factor depending on the type of shear reinforcement to be multiplied on the punching shear resistance to calculate the maximum punching shear resistance
$\eta_{sys,sb}$	enhancement factor depending on the type of shear reinforcement to be multiplied on the concrete failure criterion to obtained the failure criterion associated with the crushing of the concrete struts
ψ	rotation
ψ_E	rotation associated to the acting shear force VE
ρ_l	steel reinforcement ratio relating to the bonded tension steel (indices referring to directions)
ρ_w	ratio of the vertical shear reinforcement ratio at the investigated control perimeter
σ_{cp}	normal concrete stresses in the critical section (indices referring to directions)
σ_{sw}	average stress in the shear reinforcement intercepted by the punching cone
τ_{Ed}	acting punching shear stress in FprEN 1992-1-1:2022 [42]
$\tau_{Rd,c}$	design punching shear stress of members without shear reinforcement in FprEN 1992-1-1:2022 [42]
$\tau_{Rd,cs}$	design punching shear stress for failures within the shear-reinforced area in FprEN 1992-1-1:2022 [42]