Disponible en www.hormigonyacero.com
Hormigón y Acero 2019; 70(288): 21-33
https://doi.org/10.33586/hya.2019.001

Implementation of buckling in the simplified method of EC4-1-1 for CFT sections

Implementación del efecto de pandeo en el método simplificado del EC4-1-1 para secciones tubulares mixtas

Albert Albareda-Valls*, Jordi Maristany Carreras, Lucrecia Calderón Valdiviezo, Carlos Muñoz Blanc

Polytechnic University of Catalunya, UPC, Department of Technology in Architecture, Diagonal 649, Barcelona, 08028, Spain

Recibido el 6 de enero de 2018; aceptado el 5 de octubre de 2018

ABSTRACT

Eurocode 4 about composite structures proposes a simplified method to design concrete-filled tube sections subjected to combined compression and bi-axial bending, as an alternative to the general one. This method is based on determining the validity of a section by comparing the acting forces with the interaction curve N–M. In order to take slenderness into account, a reduction factor χ is defined, leading to a reduction of the compression and bending strength. This text proposes a new approach based on the simplified method mentioned above, thought to provide designers a faster and simpler checking and designing method for this typology of sections, including also buckling effects. With this purpose, a polynomial function is proposed depending on five known parameters: χ, χ_d, χ_{pm}, r and μ_{max}, for which a new formulation is presented.

KEYWORDS: Composite columns; Concrete filled tubes; Buckling; Eurocode; Interaction diagram

RESUMEN

El Eurocódigo 4 sobre estructura mixta propone un método simplificado para diseñar secciones tubulares mixtas sometidas a estados combinados de compresión y flexión, como alternativa al método analítico general. Este método se basa en determinar la validez de una sección mediante la comparación de la combinación de fuerzas actuantes con el correspondiente diagrama de interacción N-M de la sección. Con el objetivo de considerar la esbeltez en la resistencia de la pieza, se introduce un factor reductor de la resistencia, χ, que afecta la respuesta a compresión y también flexión combinada. Este texto propone la integración del coeficiente en una nueva aproximación numérica basada en el método simplificado, pensada para facilitar a arquitectos e ingenieros la comprobación y diseño de este tipo de secciones. Con este objetivo, se propone una función polinómica dependiente de cuatro parámetros conocidos χ, χ_d, χ_{pm}, r y μ_{max}, las ecuaciones de los cuales se detallan también a continuación.

PALABRAS CLAVE: Pilares mixtos; Secciones tubulares mixtas; Pandeo; Eurocódigo; Diagrama de interacción

1. APPROACH TO THE SIMPLIFIED METHOD PROPOSED BY EC-4

Concrete-filled tube sections are being increasingly use due to their structural performance, especially for their seismic behavior. Besides, these sections simplify considerabl the construction process and allow certain high-rise construction methods for tall buildings [1]. However, the design and validity process proposed by the European Standard (EC4) for this type of sections is not so simple than their execution. This paper pretends to bring more simplicity to the validity process, oriented to slender columns subject-

* Corresponding author.
E-mail address: albert.albareda@upc.edu (A. Albareda-Valls).

https://doi.org/10.33586/hya.2019.001

Nomenclature

X	buckling coefficient
A	section area
β	boundary conditions factor
D	diameter
b	section width
δ	steel contribution ratio
e	eccentricity
(EI)e	effective flexural stiffness
E	Young modulus
f	strength
h	distance to the centre-line
I	moment of inertia
L	length
M	bending moment
N	axial force
R	section radius
r	ratio of the smaller to the larger end moment
t	thickness
W	section modulus
μ	non-dimensional moment
χ	non-dimensional axial force
λ	relative slenderness
γ	safety factor

Subscripts

y	steel
c	concrete
s	reinforcement
pm, pc	corresponding value to concrete core
pa	corresponding value to steel tube
ps	corresponding value to reinforcement bars
pcn	corresponding value to concrete in a $2h_n$ area
pan	corresponding value to steel tube in a $2h_n$ area
psn	corresponding value to reinforcement in a $2h_n$ area
k	characteristic value
d	design value [reduced by safety factor]
Ecm	secant modulus of elasticity for concrete
R	resisted value
S	applied value
p, pl	plastic
cr	Euler critical load
n	neutral axis
max	maximum value

ed to combined compression and bending. Eurocode 4 for composite structures UNE-EN 1994-1-1:2011 [2] proposes two different methods to check and design composite columns (for sway and non-sway columns): the general and the simplified method.

While the general method requires specific software to check any geometry of composite sections, the second one is especially thought to be easily used manually by following simple expressions proposed in the European codes. This second approach shows clear restrictions and requires the section to satisfy a list of conditions.

Although the simplified method of EC4-1-1 pretends to be fast and simple to use, the truth is that the need of drawing one or two interaction diagrams every time slows considerably the process and leads the manual checking to a non-operative methodology. This text pretends to provide designers with some new practical expressions derived from the mentioned simplified approach, in which the strength decrement by buckling effects is also implemented in the compression and bending interaction diagram. Since new expressions are based on this simplified method, the restrictions are the same as those established by the European codes. Shear force effects should be taken into account only in case that the acting shear force on the steel section is higher than the 50% of the shear resisted force by the tube, as is considered by the Eurocode 4.

This article refers exclusively to circular and rectangular concrete-filled tube sections, composed by an outer steel tube and a concrete filling inside (with or without reinforcement bars). The proposed formulation is limited by the restrictions of the simplified method of EC-4, although other codes establish different more restrictive limits [3–6].

1.1. Consideration of buckling

Flexural stability of a composite column within a structure may be checked by following 3 different methods, according to the UNE-EN 1994-1-1:2011 [2]:

a) By global analysis of the structure taking second-order-moments and global imperfections into account. The validation of the cross-section has to be done by means of the simplified method of EC-4.
b) By individual analysis of the member, considering end-moments and forces from global analysis, including second-order effects and global imperfections (when relevant),through coefficients k and β. The validation of the cross-section has to be also done by means of the simplified methodof EC-4.
c) By using the buckling curves (for compressed elements) in order to consider second-order effects and member imperfections. This verification should take into account end forces and moments of the structure, including global imperfections and second-order moments when relevant. To use bucklingcurves it is needed to use the buckling length of the element,equivalent to the system length.

In the simplified procedure which is proposed here, the stability of the column is taken into account by using the last option, which is based on the European buckling curves. These curves provide a reduction factor (χ) of the compressive strength of the member, depending on the structure and slenderness. In this way, second-order moments coming from local imperfections are considered by this parameter, by reducing the axial capacity of the column. Needless to say that second-order effects coming from global imperfections and global geometry of the structure should be also considered apart by end moments and forces to the member.

Thus, the final compressive strength affected by the buckling effects can be obtained through the following expression:

$$N_{Rd} = \chi \, N_{pl,Rd} \tag{1}$$

In order to calculate the value of parameter χ for each particular case, it will be necessary to previously find out certain geometrical and resistant parameters of the column. The first of these parameters is the relative slenderness ratio of the column.

1.2. Parameter χ.

The relative slenderness ratio can be obtained by:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,R}}{N_{cr}}} \tag{2}$$

where the numerator is the plastic compressive strength [in characteristic values], and the denominator is the value of the Euler critical load.

$$N_{pl,Rd} = A_c f_{ck} + A_a f_{yk} + A_s f_{sk} \tag{3}$$

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{(L\,\beta)^2} \tag{4}$$

where $(EI)_{eff}$ is the effective stiffness of the section and β the boundary condition factor proposed by EC4-1-1 [2]. In a conservative way, a value of 1.0 can be used for non-sway columns. The effective stiffness of a composite circular and rectangular tubular section is:

$$(EI)_{eff} = E_a I_a + E_s I_s + K_c E_{cm} I_c \tag{5}$$

where coefficient K_c is 0.60 and I_a , I_s , and I_c are the second moments of area of components.

To take long-term effects on the flexural stiffness of a section into account, the modulus of stiffmess of concrete E_{cm} should be reduced to the value $E_{c,eff}$, defined by EC according to the following expression:

$$E_{c,eff} = E_{cm} \frac{1}{1+(N_{G,Ed} / N_{Ed})\, \varphi t} \tag{6}$$

being φt the creep coefficient according to 5.4.2.2.

Then, using the obtained λ, coefficient χ can be calculated from the following expressions:

$$\chi = \frac{1}{\phi + \sqrt{\phi - \bar{\lambda}^2}} \tag{7}$$

$$\phi = 0.5\,[1 + a\,(\bar{\lambda} - 0.2) + \bar{\lambda}^2] \tag{8}$$

The factor α depends on the buckling curve used for each type of composite section [Table 1]. According to EC-4, circular and rectangular concrete filled tubes with a reinforcement ratio up to 3% of concrete area require a value of 0.21, corresponding to curve "a". The rest with reinforcement ratios between 3 and 6%, require a coefficient of 0.34 from buckling curve "b" of Eurocodes, as any other composite section [6].

TABLA 1
European buckling curves.

Curve a	Circular and rectangular concrete filled tubes $A_s \leq 3\%$
Curve b	Circular and rectangular concrete filled tubes $6\% \geq A_s \leq 3\%$

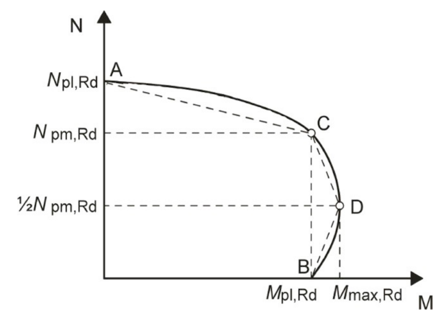

Figure 1. Interaction diagram N–M from four basic points

1.3. Squash or plastic compressive resistance

The maximum compressive or load (or "*squash load*") resisted by a composite section is defined in EC4-1-1 through the following expression:

$$N_{pl,Rd} = \frac{Aa\,f_{yk}}{\gamma_a} + \frac{Aa\,f_{sk}}{\gamma_s} + \frac{A_c f_{ck}}{\gamma_c} \tag{9}$$

For circular concrete-filled tubes, the European code allows to increase their compressive strength as a consequence of the confinement effect developed by the steel tube over the core. Thus, the ultimate resistance may increase up to 15% over the nominal one, according to the expression:

$$N_{pl,Rd} = Aa\,\eta_a\,f_{yd} + A_s\,f_{sd} + A_c\,f_{cd}\left[1 + \eta_c\,\frac{t}{d}\,\frac{f_y}{f_{ck}}\right] \tag{10}$$

Coefficients η_a and η_c proposed in the Eurocode define the variation of strength in steel and in concrete. The former refers to the decrement of resistance in steel as a consequence of the bi-axial tensional state, and the latter refers to the increment of compressive strength in concrete due to the tri-axial state. These coefficients are defined by [1] as:

$$\left.\begin{array}{l} \eta_c = \eta_{c0}\left(1 - \dfrac{10e}{d}\right) \\ \eta_a = \eta_{a0} + (1-\eta_{a0})\,\dfrac{10e}{d} \end{array}\right\} \; 0 < \frac{e}{d} < \frac{1}{10} \tag{11}$$

$$\eta_{c0} = 4.9 - 18.5\,\bar{\lambda} + 17\,\bar{\lambda}^2 \;;\; \eta_{c0} \geq 0 \tag{12}$$

$$\eta_{a0} = 0.25\,(13 + 2\,\bar{\lambda}) \;;\; \eta_{a0} \leq 1{,}0 \tag{13}$$

1.4. Combined compression and bending resistance

The resistance of a concrete-filled steel tube subjected to compression and bending, according to Eurocode 4, can be obtained by drawing one interaction diagram N–M for each axis of the section, by using four singular points. These four points come from different positions of the neutral axis [Fig. 1].

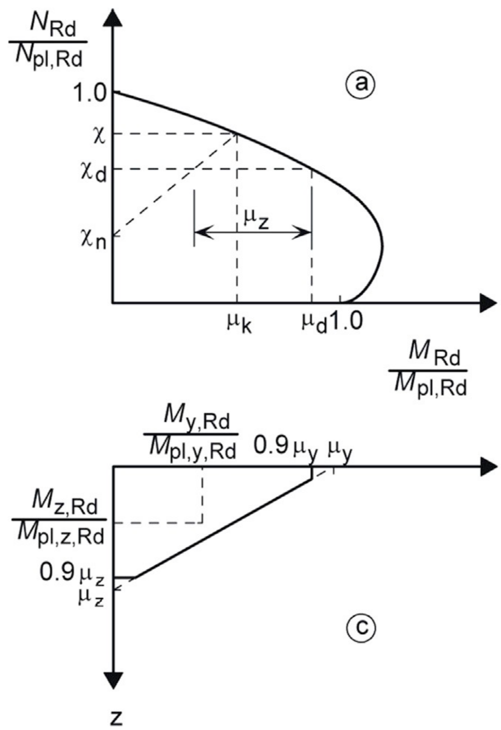

Figure 2. Interaction diagram M_y – M_z [2].

In order to calculate the squash load and the bending resistance of a section at points A, B, C and D, different situations have to be considered:

Point A $NA = N_{pl,Rd}$
$MA = 0$
Point B $NB = 0$
$MB = M_{pl,Rd}$
Point C $NC = N_{pm,Rd}$
$MC = M_{pl,Rd}$
Point D $ND = N_{pm,Rd}/2$
$MD = M_{max,Rd}$

where the plastic moment resistance of the section $M_{pl,Rd}$ is:

$$M_{pl,Rd} = f_{yd}\,(W_{pa} - W_{pan}) + 0.5\, f_{cd}\,(W_{pc} - W_{pcn}) + f_{sd}(W_{ps} - W_{psn}) \quad (14)$$

W_{pc} and W_{pa} are the plastic section modules for the concrete core and the steel tube respectively, and W_{pcn} and W_{pan} the plastic section modules for both in the area limited by a distance of $2h_n$, being h_n the distance of the neutral axis from the centroid.

The maximum bending moment resisted by a composite section, when it is also subjected to axial load, is [Moment in D]:

$$M_{max,Rd} = f_{yd}\;W_{pa}\; + 0.5\, f_{cd}\;W_{pc}\; + f_{sd}\;W_{ps} \quad (15)$$

To design a concrete-filled tube under combined compression and bi-axial bending, it is necessary to check the validity of the section in both axes. According to Eurocode, imperfections should be only considered in the plane which failure is expected to occur; in case of being ambiguous, both axes must be checked.

EC4-1-1 limits the non-dimensional moment to a ratio a_M, which is 0.9 for steel grades S235 and S355 and 0.8 for steel grades S420 and S460 in case of uni-axial compression (steel and to 1.0 in case of bi-axial compression [Eqs. (16)–(19)] [Fig. 2].

$$\frac{M_{y,Sd}}{\mu_y\, M_{pl,Rd}} \le a_M \quad (16)$$

$$\frac{M_{z,Sd}}{\mu_z\, M_{pl,Rd}} \le a_M \quad (17)$$

$$\frac{M_{y,Ed}}{\mu_y\, M_{pl,Rd}} + \frac{M_{z,Ed}}{\mu_z\, M_{pl,Rd}} \le 1.0 \quad (18)$$

$$\mu_y = \frac{M_{y,Ed}}{a_M\, M_{pl,Rd}} \quad (19)$$

where the non-dimensional moment resisted by a section is μ [considering buckling effects], obtained from χ and χ_n. The value χ_n is defined below by the European standards for concrete-filled tubes as follows, depending on parameter r (the ratio of the smaller to the larger end moment of the column):

$$\chi_n = \frac{(1-r)}{4}\,\chi \;\; \text{for} \;\; \bar{\lambda} \le 2.0$$

$$r = \frac{M_{max}}{M_{min}} \quad (20)$$

The influence of shear on the resistance to combined compression and bending forces should be taken into account when determining the interaction curve if the shear force $V_{a,Ed}$ on the steel section exceeds 50% of the design shear resistance $V_{a,pl,Ed}$. Under this assumption, the interaction of shear should be considered by reducing the design steel strength by $(1 - p)\, f_{yd}$ in the shear area, according to UNE-EN 1994-1-1:2011.

2.
MECHANICAL PROPERTIES OF THE COMPOSITE SECTION

2.1. Determination of neutral axis

To calculate the maximum bending strength of a concrete-filled tube section by following the proposal of the simplified method of UNE-EN 1994-1-1:2011 [2], it is necessary first to determine the neutral axis of the cross-section.

In this way, by equating tension and compression forces from position of neutral axis h_n [Fig. 3], it is derived that the

$$F_C = F_{1,a} \quad (21)$$

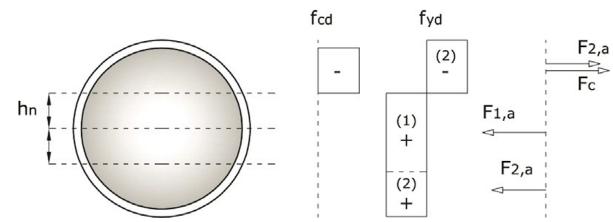

Figure 3. Determination of h_n in circular sections.

compressive force generated by concrete above this axis must be equal to the traction force of steel tube ($F_{1,a}$) within a region limited by a distance of $2h_n$ from the centroid:

$$F_c = F_{1,a} \tag{21}$$

The Appendix C of UK National Annex to Eurocode 4 [7] proposed an approximate expression to calculate the position of neutral axis for circular and rectangular concrete-filled tube sections, with a 15% maximum deviation [Eq. (22)], since it is very difficult to calculate the exact value.

$$h_n = \frac{N_{pm} - A_{sm}\,(2f_{fsd} - f_{sd})}{2d\,f_{cd} + 4t\,(2f_{yd} - f_{cd})} \tag{22}$$

2.2. Determination of plastic section module

In circular sections, the plastic section modulus of the core can be obtained by:

$$W_{pc} = \frac{4}{3}(R-t)^3 \tag{23}$$

while the plastic section modulus of steel tube by:

$$W_{pa} = \frac{4}{3}[R^3 - (R-t)^3] \tag{24}$$

Within the area limited by a distance $2h_n$ from the centroid of the section, the plastic section modulus of the core can be obtained according the following expression:

$$W_{pcn} = \frac{4}{3}\left[(R-t)^3 \sqrt{[(R-t)^2 - h_n^2]^3}\right] \tag{25}$$

while regarding to the tube:

$$W_{pac} = \frac{4}{3}\left[R^3 - \sqrt{(R-h_n^2)^3} - (R-t)^3 + \sqrt{[(R-t)^2 - h_n^2]^3}\right] \tag{26}$$

Repeating the same analysis for rectangular sections, the following expressions are obtained:

$$W_{pcn} = \frac{(B-t)-(H-t)^2}{4} \tag{27}$$

$$W_{pa} = \frac{BH^2 - (B-t) - (H-t)^2}{4} \tag{28}$$

$$W_{pcn} = \frac{(B-t) - (h_n - t)^2}{4}$$

$$W_{pan} = \frac{B\,h_n^2 - (B-t) - (h_n - t)^2}{4} \tag{29}$$

2.3. Determination of the plastic bending resistance

According to UNE-EN 1994-1-1:2011 [2], the plastic bending moment which can be resisted by a section at point B of its interaction diagram can be written as:

$$M_{p,Rd} = f_{yd}\,(W_{pa} - W_{pan}) + 0.5\,f_{cd}\,(W_{pc} - W_{pcn}) + f_{sd}\,(W_{ps} - W_{psn}) \tag{30}$$

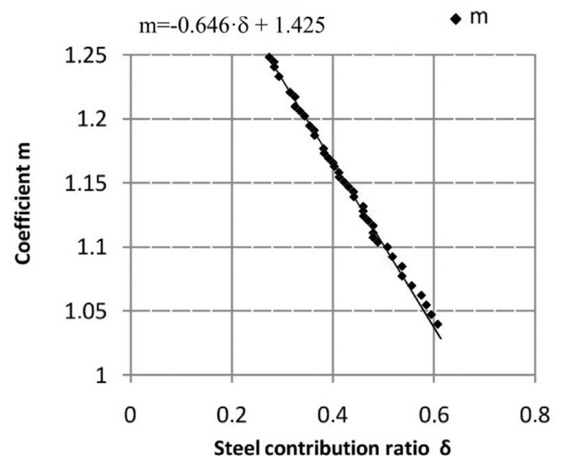

Figure 4. Coefficient m for different values of δ in circular sections. Values from [8].

By using the tables published by CIDECT in the Design Guide no. 5 [8] (Fig. 4), obtained from a wide regression of results, some analytic expressions are proposed in order to calculate the plastic moment resistance of a section, needless to draw its neutral axis before. In this way, for circular concrete-filled tubes, the plastic moment resistance can be quickly obtained through the following expression:

$$M_{pl,Rd} = m_{\odot}\,\frac{d^3 - (d-2t)^3}{6}\,f_{yd} \tag{31}$$

where the value of m can be calculated using Table 11 of the Design Guide of CIDECT:

$$m_{\odot} = -0.646\,\delta + 1.425 \tag{32}$$

For circular sections, depending on a linear regression of results from sections with different diameters, thicknesses and material strengths:

$$m_{\odot} = -0.402\,\delta + 1.342 \tag{32}$$

For rectangular and square-shaped sections, the plastic moment resistance can be obtained from [8]:

$$M_{pl,Rd} = m_{\odot}\,\frac{h^2 b - (h-2t)^2 (b-2t)}{4}\,f_{yd} \tag{34}$$

From Table 9 of the same design guide [8], the following diagram is obtained for rectangular sections:
The following expression is obtained from a linear regression of the diagram in Fig. 5:

$$m_{\odot} = -0.402\,\delta + 1.342 \tag{35}$$

Using the same methodology for rectangular sections of different slenderness $h/t = 0.5$ and $h/t = 2$, the value of m can be calculated by two linear equations depending on δ:

$$m_{\odot}\left(\frac{h}{t} = 0.5\right) = -0.235\,\delta + 1.215 \tag{36}$$

$$m_{\odot}\left(\frac{h}{t} = 2\right) = -0.568\,\delta + 1.475 \tag{37}$$

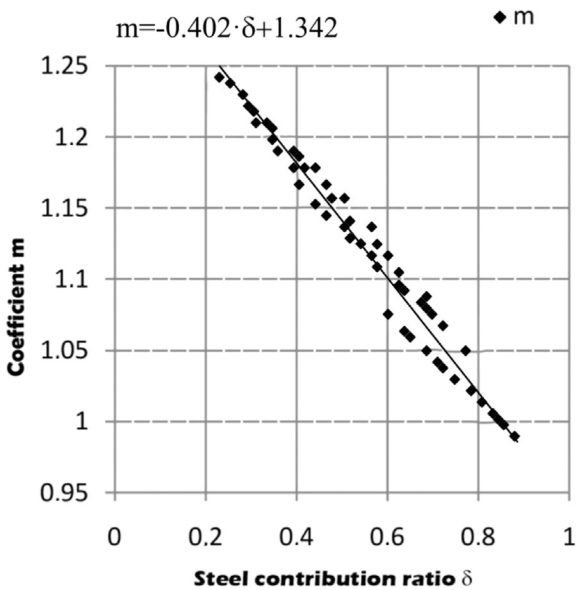

Figure 5. Coefficient *m* for different values of δ in rectangular sections. Values from [8].

In case of having reinforcement bars inside the concrete core, its contribution in the bending response should be added to the general plastic moment resistance of the composite section. The plastic moment resistance of the reinforcement can be obtained through [Eq. (38)].

$$M_{pl,s,Rd} = \sum_{i=1}^{n} |e_i| \, A_{si} \, f_{sd} \tag{38}$$

with:

A_s : area of reinforcement bars.

$|e_i|$: distance of the bar to the centre-line of the section.

The increase of plastic moment resistance provided by reinforcement can be approximated by different expressions, depending on the general shape of the section:

$$\Delta M_{pl} = M_{pl,s,Rd} \tag{39}$$

For circular sections:

$$M_{pl,s,Rd} = \frac{(r-30)^2 - 8\,[(r-30)^2 - p\,Ac/\pi]^{3/2}}{6} \, f_{sd} \tag{40}$$

where $r = R - t$, and supposing that reinforcement is uniformly distributed around the perimeter of the section.

For rectangular or square sections:

$$M_{pl,s,Rd} = \frac{\rho \, A_c \,(h - t - 30)}{4} \, f_{sd} \tag{41}$$

h is the height of the section, and ρ its reinforcement ratio:

$$\rho = \frac{A_s}{A_c} \tag{42}$$

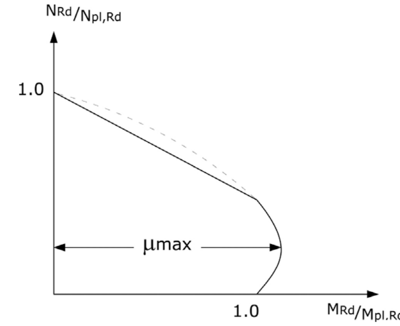

Figure 6. Maximum moment resistance.

TABLE 2
Values of mmax depending on δ.

δ	μ_{max}	μ_{max} (1)	μ_{max} (0.5)	μ_{max} (2)
0.20	1.800	1.810	1.815	1.840
0.25	1.580	1.601	1.610	1.610
0.30	1.430	1.450	1.450	1.460
0.35	1.330	1.330	1.340	1.320
0.40	1.260	1.250	1.260	1.280
0.45	1.200	1.200	1.200	1.220
0.50	1.150	1.150	1.150	1.180
0.55	1.120	1.120	1.120	1.130
0.60	1.100	1.100	1.100	1.110
0.65	1.090	1.090	1.090	1.100
0.70	1.070	1.070	1.070	1.080
0.75	1.050	1.050	1.050	1.070

2.4. Determination of the maximum moment resistance ratio

According to the previous section of this paper, the plastic moment resistance of a section can be obtained by [Eq. (30)]. Contrarily, the maximum bending moment resistance in presence of axial force [with a value of load equal to 50% of the compressive strength of concrete core] is:

$$M_{max} = f_{yd} \, W_{pa} + 0.5 \, f_{cd} \, W_{pc} + f_{sd} \, W_{ps} \tag{43}$$

Then, the non-dimensional maximum moment resistance ratio referred to $M_{pl,Rd}$ will be (Fig. 6):

$$\mu_{max} = \frac{M_{max}}{N_{pl,Rd}} \tag{44}$$

The maximum moment resistance ratio for concrete filled steel tube sections can also be obtained directly by means of the following Table 2 depending on δ.

$$\delta = \frac{A_a \, f_{yd}}{N_{pl,Rd}} \tag{45}$$

where : $0.2 \le \delta \le 0.9$

Alternatively, μ_{max} can be calculated by a polynomial equation (46) obtained from the values shown in Table 2, for any shape of concrete filled tubes (Fig. 7).

$$\mu_{max} = -5.144\,\delta^3 + 10.77\,\delta^2 - 7.657\,\delta + 2.916 \tag{46}$$

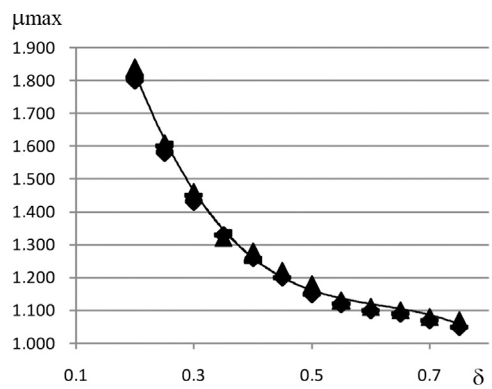

Figure 7. Values of μ_{max} depending on δ.

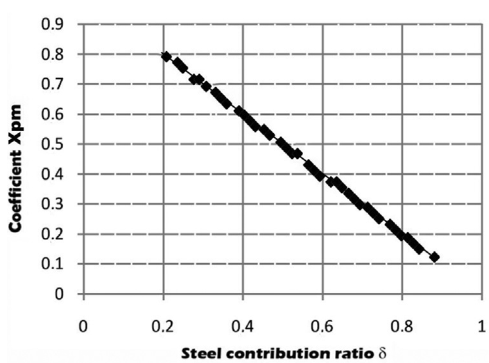

Figure 8. Values of χ_{pm} depending on δ for rectangular sections.

Curiously, μ_{max} depends only on the steel contribution ratio and the shape of the tube has almost no influence on this value. The third degree polynomial equation is also valid for reinforced sections, provided that its contribution can be integrated in the value of δ, $N_{pl,Rd}$ and $M_{pl,Rd}$.

μ_{max} is a determining variable of the composite section behavior, as will be shown later in the new formulation proposed in this paper.

2.5. Determination of non-dimensional compressive strength of concrete core

One of the parameters defined by the simplified method of EC-4 [2] is the non-dimensional compressive strength of the core. It corresponds to the ratio between the compressive strength of the core under compression and the squash load of the global composite section. This value can be defined analytically:

$$\chi_{pm} = \frac{A_c f_{cd}}{N_{pl,Rd}} \tag{47}$$

A linear diagram can be obtained, from an analysis of several cases of different diameters, tube thicknesses and material strengths, as it is shown in Fig. 8:

$$\chi_{pm} = 1 - \delta \tag{48}$$

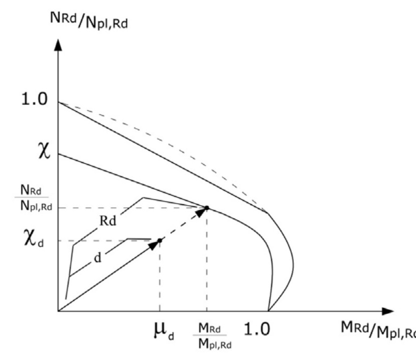

Figure 9. Methodology of checking validity of a section.

3.
NEW APPROACH PROPOSED

3.1. Sectional design from the simplified method of EC-4

According to the simplified method proposed in EC4-1-1, a concrete-filled tube section is valid under combined compression and bending in case of satisfying the interaction diagram area. For this purpose, the European code proposes a specific methodology to determine this diagram from four known points, result of different positions of the neutral axis – as mentioned in Section 1 of this paper.

A practical consequence of this process is the need of drawing an interaction diagram N–M for every particular case, with the aim of obtaining the maximum moment that the section can resist, depending on the percentage of axial load.

The process is more complicated when dealing with slender columns, when the ultimate compressive load is sensibly lower than the squash load of the section. Then, the interaction diagram defined by four points of EC4 is no longer valid and it is necessary to redraw a new function, by implementing a corresponding buckling reduction.

The new methodology dealt with in this paper proposes an analytical approximation to this process, by defining a new function corresponding to the interaction diagram of a section with buckling effects directly incorporated. This way, the new methodology proposes to compare the acting non-dimensional bending moment with the maximum moment resistance of the section, depending on load percentage. The section is valid when the module of the acting vector (*d*) is lower than the module of the resisting vector (Rd).

See Fig. 9.

The new methodology proposed here tries to optimize the process of checking validity by means of suppressing the drawing of interaction diagrams and the implementation of buckling effects.

3.2. New design approach

The new approach is based on defining an approximate function, capable of reproducing the reduced interaction diagram of a composite section:

$$\mu_d = f(\chi_d) \tag{49}$$

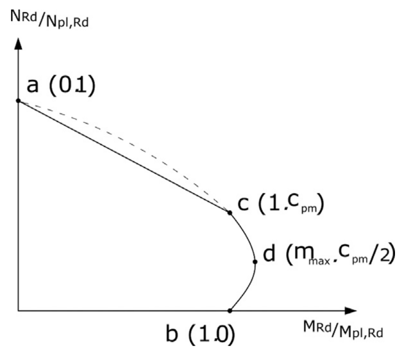

Figure 10. Simplified interaction diagram.

With this purpose, function *f* has been defined in two different parts: one linear between a–c, and the second parabolic between c–d–b. In zone a–c, and according to British Standards [7], the interaction diagram can be approximated by a linear equation:

$$\mu_d = B\,\chi_d + C \tag{50}$$

And in part c–d–b, the diagram can be replaced by a parabolic equation according to [Fig. 10]:

$$\mu_d = A'\,\chi_d^2 + B'\,\chi_d + C' \tag{51}$$

with this assumption, coefficients *A*, *B* and C can be obtained from four known points:

$a = (0, 1) \quad b = (1, 0)$

$c = \; 1, \chi_{pm} \quad d = \; (\mu_{max} \;, \; \chi_{pm}/2)$

The coefficients of the linear equation are:

$$A = 0 \tag{52}$$

$$B = \frac{1}{\chi_{pm} - 1} \tag{53}$$

$$C = \frac{-1}{\chi_{pm} - 1} \tag{54}$$

And the coefficients of the parabolic equation are:

$$A' = \frac{4\,(\mu_{max} - 1)}{-\,(\chi_{pm})^2} \tag{55}$$

$$B' = \frac{4\,(\mu_{max} - 1)}{-\,(\chi_{pm})^2} \tag{56}$$

$$C' = 1 \tag{57}$$

Considering these coefficients, a new function depending on three variables is obtained:

$$\mu_d = f\,(\chi_d, \chi_{pm}, \mu_{max}) \tag{58}$$

By replacing coefficients *A*, *B* and C in the original function for zone a–c:

$$\mu_d = \left[\frac{1}{\chi_{pm} - 1}\right] \chi_d + \left[\frac{-1}{\chi_{pm} - 1}\right] \tag{59}$$

In the same way, parabolic equation in zone c–d–b is:

$$\mu_d = \left[\frac{4\,(\mu_{max} - 1)}{-\,(\chi_{pm})^2}\right] \chi_d^2 + \left[\frac{4\,(\mu_{max} - 1)}{\chi_{pm}}\right] \chi_d + 1 \tag{60}$$

3.3. *Implementation of buckling effects*

When calculating the maximum moment resistance ratio of a section μ for a specific axial load percentage χ_d , it is necessary to consider buckling effects.

According to Fig. 2 from European Standards EC4-1-1, the second-order moment resistance ratio for slender columns is:

$$\mu_k = \frac{\chi - 1}{\chi_{pm} - 1} \tag{61}$$

That is the moment resistance ratio of a section for a certain axial load ratio χ, which is just the maximum moment resisted by a section depending on its slenderness and according to buckling curves from EC-3.

$$\mu' = \frac{(\chi_d - \chi_n)}{(\chi - \chi_n)} \; \frac{(\chi - 1)}{(\chi_{pm} - 1)} \tag{62}$$

For concrete-filled steel tubes, the lower limit from which second-order moments do not have to be considered:

$$\chi_n = \frac{(1 - r)\,\chi}{4} \quad \text{for } \lambda \leq 2.0 \tag{63}$$

Introducing Eq. (63) into Eq. (62):

$$\mu_k = \frac{[4\,\chi_d - (1 - r)\,\chi]}{[(3 + r)\,\chi]} \; \frac{\chi - 1}{\chi_{pm} - 1} \;\rightarrow$$

$$\mu_k = \frac{4\,(\chi - 1)}{(3 + r)\,(\chi_{pm} - 1)\,\chi}\,\chi_d - \frac{(1 - r)\,(\chi - 1)}{(3 + r)\,(\chi_{pm} - 1)} \tag{64}$$

Being *r* the ratio of the smaller to the larger end moment, according to chapter 4.8.3.13 of EC4-1-1.

This way, subtracting Eq. (64) from Eq. (59):

$$\mu_d = \left[\frac{1}{\chi_{pm} - 1}\right] \left[1 - \frac{4\,(\chi - 1)}{(3 + r)\,\chi}\right] \chi_d + \left[\frac{1}{\chi_{pm} - 1}\right] \left[1 - \frac{(1 - r)\,(\chi - 1)}{(3 + r)}\right] \tag{65}$$

And applying the same operation to Eq. (59):

$$\mu_d = \left[\frac{4\,(\mu_{max} - 1)}{-\,(\chi_{pm})^2}\right] \chi_d^2 + \left[\frac{4\,(\mu_{max} - 1)}{-\,(\chi_{pm})^2} - \frac{4\,(\chi - 1)}{(3 + r)\,(\chi_{pm} - 1)\,\chi}\right] \chi_d + \left[1 + \frac{(1 - r)\,(\chi - 1)}{(3 + r)\,(\chi_{pm} - 1)}\right] \tag{66}$$

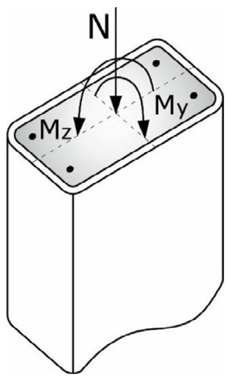

Figure 11. Combined compression and bending state.

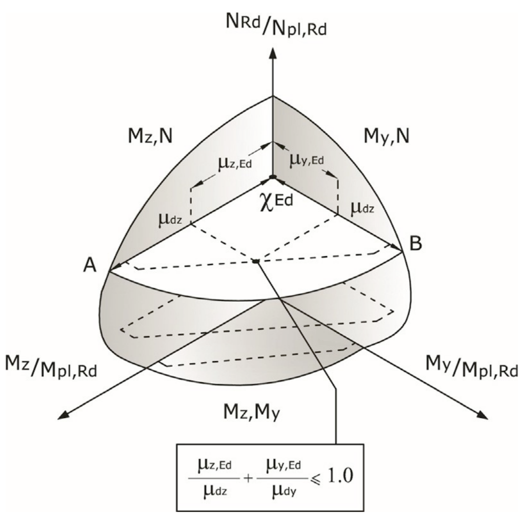

Figure 12. Interaction diagram $N - M_y - M_z$.

Consequently, the proposed function which has to be coincident with reduced interaction diagram, depends on five known variables:

$$\mu_d = f\left(\chi_d , \chi_{pm} , \chi, \mu_{max} , r\right) \quad (67)$$

3.4. *Combined compression and bending*

For combined compression and bending [Fig. 11], and according to chapter 1.4 of this text, EC4-1-1 proposes an interaction diagram for each axis μ_y , μ_z depending on the value of the non-dimensional axial force χ_{Ed} .

It will be necessary to check the section with a lower percentage of strength due to buckling effects only in its minor axis [according to EC4-1-1]. This way, a tridimensional validity surface is generated by composing the interaction diagrams in the two axes [Fig. 12], one of them reduced by buckling effects.

In order to check a circular section subjected to combined bi-axial bending and compression, a combination of moments in the two axes will be enough. The following function may be satisfied:

$$\sum \mu_{Ed} , \leq f\left(\chi_d , \chi_{pm} , \chi, \mu_{max} , r\right) \quad (68)$$

In particular:

$$\sum \mu_{,Ed} \leq A\,\chi_d^2 + B\,\chi_d + C \quad (69)$$

For circular sections:

$$\sum \mu_{,Ed} = \sqrt{\mu_{y,Ed^2} + \mu_{z,Ed^2}} \quad (70)$$

The validity process for square and rectangular sections is quite different from the one for circular sections, due to the presence of two different axes. In these cases, and according to EC4-1-1, it will be only necessary to check the section with buckling effects implemented in its minor axis. In the approach which is proposed in this paper, an extra coefficient D is defined in order to describe the proportion between bending moments with and without buckling effects in their respective axes. General condition for validity is:

$$\frac{\mu_{y,Ed}}{A_y\,\chi_d^2 + B_y\,\chi_d + C_y} + \frac{\mu_{y,Ed}}{A_z\,\chi_d^2 + B_z\,\chi_d + C_z} \leq 1 \quad (71)$$

$\mu_{z,Ed}$ is considerated as the acting moment in the major axis [in this axis there is no need to consider buckling effects]. The ratio between these two non-dimensional moments [coefficient D] is:

$$\frac{A_y\,\chi_d^2 + B_y\,\chi_d + C_y}{A_z\,\chi_d^2 + B_z\,\chi_d + C_z} + \frac{[\text{With buckling in } Y]}{[\text{With buckling in } Z]} \quad (72)$$

For the second interval $\chi_{pm} > \chi_d >$

$$D = \left[\frac{\left(A\,\chi_d^2 + B\,\chi_d + C\right)\chi_{pm}{}^2}{4\,(1-\mu_{max})\,\chi_d\,(\chi_d - \chi_{pm}) + \chi_{pm}{}^2}\right] \quad (73)$$

Resulting in:

$$\frac{\mu_{y,Ed}}{[A_y\,\chi_d^2 + B_y\,\chi_d + C_y]} + \frac{D\,\mu_{z,Ed}}{[A_y\,\chi_d^2 + B_y\,\chi_d + C_y]} \leq 1 \quad (74)$$

What is:

$$\mu_{y,Ed} + D\,\mu_{z,Ed} \leq \left[A_y\,\chi_d^2 + B_y\,\chi_d + C_y\right] \quad (75)$$

Accepting coefficients A_y, B_y y C_y and non-dimensional moment $\mu_{y,Ed}$, always referred to minor axis of the section.

3.5. *Proposed expressions*

This paper proposes a new methodology for checking the validity of concrete-filled tubes subjected to compression and bending, with buckling effects also implemented according to simplified method of Eurocode 4. With this purpose, a new second order polynomial function is defined; this function depends on five known variables, as mentioned before:

$$f\left(\chi_d , \chi_{pm} , \chi, \mu_{max} , r\right) = A\,\chi_d^2 + B\,\chi_d + C \quad (76)$$

Despite of this paper is oriented to concrete-filled tubes, equations given below are also valid for any other composite symmetric section, by respecting restrictions and criteria proposed in EC-4.

Defining the following variables:

$\chi, \chi_{pm}, r, \mu_{max}$

Applied compressive non-dimensional load χ_d:

$$\chi_d = \frac{N_{Sd}}{N_{pl,Rd}} \tag{77}$$

For circular concrete filled sections, the squash load $N_{pl,Rd}$ is defined in EC4 as:

$$N_{pl,Rd} = A_a\,\eta_2\,f_{yd} + A_s\,f_{sd} + A_c\,f_{cd}\left[1 + \eta_1\,\frac{t}{d}\,\frac{f_y}{f_{ck}}\right] \tag{78}$$

The non-dimensional axial force corresponding to concrete core χ_{pm} :

$$\chi_{pm} = \frac{A_c\,f_{cd}}{N_{pl,Rd}} \tag{79}$$

According to Section 2.4:
$X_{pm} = 0.994 - 1.433\,\delta$ for circular sections.
$X_{pm} = 1 - \delta$ for rectangular sections.

The reduced non-dimensional axial load for buckling effects χ [according to 1.2.1].

The ratio of the smaller to the larger end moment of the column, r:

$$r = \frac{M_{Sd\,max}}{M_{Sd\,min}} \tag{80}$$

The acting non-dimensional moments can be obtained:

$$\mu_{y,Ed} = \frac{M_{yd}}{M_{pl,Rd}} \tag{81}$$

$$\mu_{z,Ed} = \frac{M_{zd}}{M_{pl,Rd}} \tag{82}$$

where plastic moment of the section $M_{pl,Rd}$ is obtained from expressions proposed in Section 2.3 of this paper. Finally, the non-dimensional maximum moment resistance of the section, according to Eq. (46):

$$\mu_{max} = -5.144\,\delta^3 + 10.77\,\delta^2 - 7.657\,\delta + 2.916$$

Coefficients A, B and C in formulation proposed correspond always to the minor axis of the section.

Finally, a section subjected to combined compression and bending is valid in case of satisfying the following conditions:

For circular sections:

$$\mu_{,Ed} \le A\,\chi_d^2 + B\,\chi_d + C \tag{83}$$

For rectangular sections:

$$\mu_{y,Ed} + D\,\mu_{z,Ed} \le A\,\chi_d^2 + B\,\chi_d + C \tag{84}$$

With the following restrictions:

$$\mu_{y,Ed} \le 0.9\left[A\,\chi_d^2 + B\,\chi_d + C\right] \le 0.9 \tag{85}$$

$$\mu_{z,Ed} \le 0.9\,D^{-1}\left[A_y\,\chi_d^2 + B_y\,\chi_d + C_y\right] \le \frac{0.9}{D} \tag{86}$$

Where coefficients A, B, C and D are defined as:

$if\ \chi > \chi_d \ge \chi_{pm} \longrightarrow$

$$A = 0 \tag{87}$$

$$B = \left[\frac{1}{\chi_{pm}-1}\right]\left[1 - \frac{4\,(\chi-1)}{(3+r)\,\chi}\right] \tag{88}$$

$$+\left[\frac{-1}{\chi_{pm}-1}\right]\left[1 - \frac{(1-r)\,(\chi-1)}{(3+r)}\right] \tag{89}$$

$$D = (B\,\chi_d + C)\left[\frac{\chi_{pm}-1}{\chi_d-1}\right] \tag{90}$$

$if\ \chi_{pm} > \chi_d > \left[\frac{1-r}{4}\right]\chi \longrightarrow$

$$A = \left[\frac{4\,(1-\mu_{max})}{(\chi_{pm})^2}\right] \tag{91}$$

$$B = \left[\frac{4\,(\mu_{max}-1)}{\chi_{pm}} - \frac{4\,(\chi-1)}{(\chi_{pm}-1)\,\chi\,(3+r)}\right] \tag{92}$$

$$C = \left[1 + \frac{(1-r)\,(\chi-1)}{(3+r)\,(\chi_{pm}-1)}\right] \tag{93}$$

$$D = \left[\frac{(A\,\chi_d^2 + B\,\chi_d + C)\,\chi_{pm}^2}{4\,(1-\mu_{max})\,\chi_d(\chi_d-\chi_{pm})+\chi_{pm}^2}\right] \tag{94}$$

$if\ \ \chi_d > \left[\frac{1-r}{4}\right]\chi \longrightarrow$

$$A = \left[\frac{4\,(1-\mu_{max})}{(\chi_{pm})^2}\right] \tag{95}$$

$$B = \left[\frac{4\,(\mu_{max}-1)}{\chi_{pm}}\right] \tag{96}$$

$$C = 1 \tag{97}$$

$$D = 1 \tag{98}$$

The formulation proposed above do not involve shear effects in order to simplify the final expressions; to take longitudinal and transversal shear also into account [7] a reduction of thickness of the steel tube can be made as an approximate method.

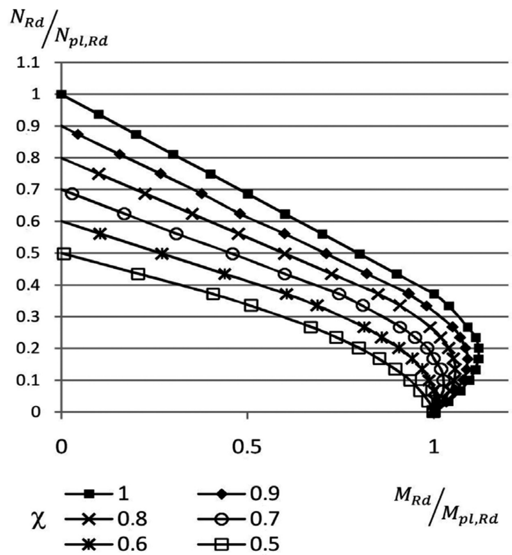

Figure 13. Interaction diagrams for different values of χ.

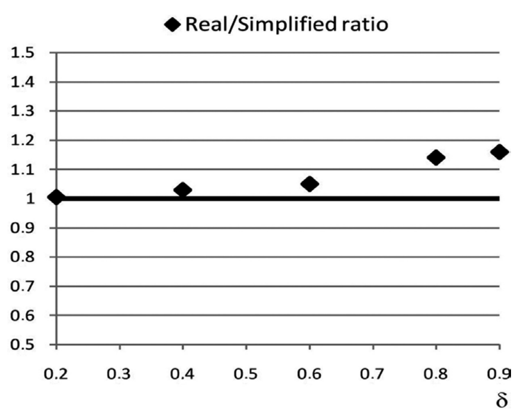

Figure 14. Deviation percentage between real and simplified diagram area, depending on δ.

4. APPLICATION CURVES

The result of applying the method proposed above is a group of interaction curves which become modified according to different values of χ, for different values of relative slenderness [Fig. 13].

Thus, this methodology provides the possibility of a simpler and quicker process to check the validity of concrete-filled steel tube section subjected to combined compression and bending, by also considering buckling effects of the column.

In Fig. 13 several interaction curves are shown for different values of χ [directly related to the slenderness of the column] for a circular concrete-filled tube specimen of 300 mm of diameter and 5 mm of wall thickness.

As mentioned before, this approach is based in a simplification of the original interaction curve N–M in two simpler equations: a parabolic and a linear one. It is important to point out that the difference between real and simplified values is quite small, and always by the side of safety.

To quantify this deviation from the original diagrams, different curves according to different steel contribution ratios have been analyzed; the higher is the steel contribution ratio of the section higher is the deviation between both curves, as is shown in Fig. 14. This graphic presents the relation between real and simplified interaction diagram area, depending on δ.

In order to verify the proposed method, a set of calculated curves have been compared with a set of experimental results by using slender and short columns from Fujinaga T., Doi H. and Sun Y.P. [8] presented in 14th World Conference on Earthquake Engineering. Specimens tested by the authors mentioned before are listed in Table 3 and named depending on L/D ratio, r end moment ratio [+1.0, +0.5, +0.0, −0.33, −0.66, −1.0, etc.], concrete strength [27 or 60 N/mm²] and eccentricity [*e*]:

Axial loads are expressed in kN and moments in kNm.

Sections tested are square-shaped [125 × 125 × 3.2 mm], made of steel STKR400 with f_y = 358 N/mm² and filled with concrete (27 and 60 N/mm² strength). Experimental results have been superimposed over new interaction diagrams derived from simplified method proposed in this paper, without applying material strength reduction coefficients. Dispersion of results is as follows in Figs. 15 and 16.

The majority of the specimens used for calibration show a good agreement between with the obtained results by means of the proposed simplified methodology. Table 3 shows the deviation of obtained values with experimental ones in terms of combined strength (or the available strength vector "d" that represents the structural capacity of the section under combined compression and bending).

The proposed method seems to be conservative, according to the comparison shown above. Most analyzed specimens resists slightly more than the strength predicted by the proposed method, as the deviation ratio is less than 1.00. Specimens that show a significant difference between the experimental results and theoretical ones are those with negative bending moment ratio (*r*); in these cases, the real tested specimens resist up to a 24% more than the predicted by the method. This is basically due to the fact that the column is subjected only to one curvature.

5. CONCLUSIONS

Simplified method proposed by the EC-4 to determine the validity of a composite section subjected to compression and bi-axial bending, leads the designer to a procedure that is far from being practical. The need of drawing two entire diagrams in order to check the validity of a section converts this process in iterative and inoperative.

This text presents a new approach which implements buckling effects of the column [according to the European buckling curves] within the interaction diagram of a concrete-filled tube section. In this way, the approach is committed to facilitate the use of a manual and simplified method and to spread this way the use of composite tubular sections.

The proposed simplification shows a good agreement with experimental results, depending on geometric param-

TABLE 3
Results of tested specimens and the corresponding obtained values by using the proposed method. The comparison is done by using the available strength vector "*d*".

Specimen	*L/D*		*r*	*Experimental values* [5]					*Obtained values*			*Deviation*
				N	*M*	*c*	*m*	*d*	*c*	*m*	*d*	*d/d*
(1)	R20-27-e-1.0(+)	20	+1.00	585	5.8	0.62	0.22	0.65	0.65	0.22	0.68	1.04
(2)		20	+1.00	440	13.2	0.47	0.45	0.65	0.52	0.52	0.73	1.12
(3)		20	+1.00	222	22.2	0.23	0.79	0.82	0.31	0.92	0.97	1.18
(4)	R20-27-e-0.5(+)	20	+0.50	627	6.3	0.67	0.22	0.70	0.65	0.21	0.68	0.97
(5)		20	+0.50	479	14.4	0.51	0.53	0.73	0.52	0.54	0.74	1.01
(6)		20	+0.50	265	26.5	0.29	0.95	0.99	0.29	0.95	0.99	1.00
(7)	R20-27-e-0.0(+)	20	+0.00	671	6.7	0.71	0.23	0.74	0.66	0.20	0.68	0.92
(8)		20	+0.00	552	16.6	0.59	0.61	0.84	0.55	0.52	0.75	0.89
(9)		20	+0.00	307	30.7	0.32	1.09	1.13	0.28	1.00	1.03	0.91
(10)	R20-27-e-0.33(−)	20	−0.33	711	7.1	0.75	0.24	0.78	0.67	0.20	0.69	0.88
(11)		20	−0.33	574	17.2	0.61	0.62	0.87	0.53	0.56	0.77	0.88
(12)		20	−0.33	328	32.8	0.35	1.17	1.22	0.28	1.02	1.18	0.96
(13)	R20-27-e-0.66(−)	20	−0.66	738	7.4	0.79	0.26	0.83	0.66	0.21	0.69	0.83
(14)		20	−0.66	620	18.6	0.66	0.66	0.93	0.55	0.56	0.78	0.83
(15)		20	−0.66	325	32.5	0.35	1.17	1.22	0.31	1.03	1.07	0.87
(16)	R20-27-e-1.00(−)	20	−1.00	823	8.2	0.88	0.27	0.92	0.67	0.21	0.70	0.76
(17)		20	−1.00	665	19.9	0.71	0.73	1.01	0.55	0.55	0.77	0.76
(18)		20	−1.00	336	33.6	0.36	1.19	1.24	0.29	1.08	1.11	0.89
(19)	R10-27-e-1.0(+)	10	+1.00	606	18.2	0.64	0.64	0.90	0.61	0.60	0.85	0.94
(20)		10	+1.00	281	28.1	0.30	1.00	1.04	0.32	1.03	1.07	1.02
(21)	R10-27-e-0.5(+)	10	+0.50	650	19.5	0.69	0.70	0.98	0.60	0.60	0.84	0.85
(22)		10	+0.50	328	32.8	0.36	1.17	1.22	0.30	1.04	1.08	0.88
(23)	R10-27-e-0.0(+)	10	+0.00	695	20.9	0.73	0.76	1.05	0.61	0.62	0.86	0.81

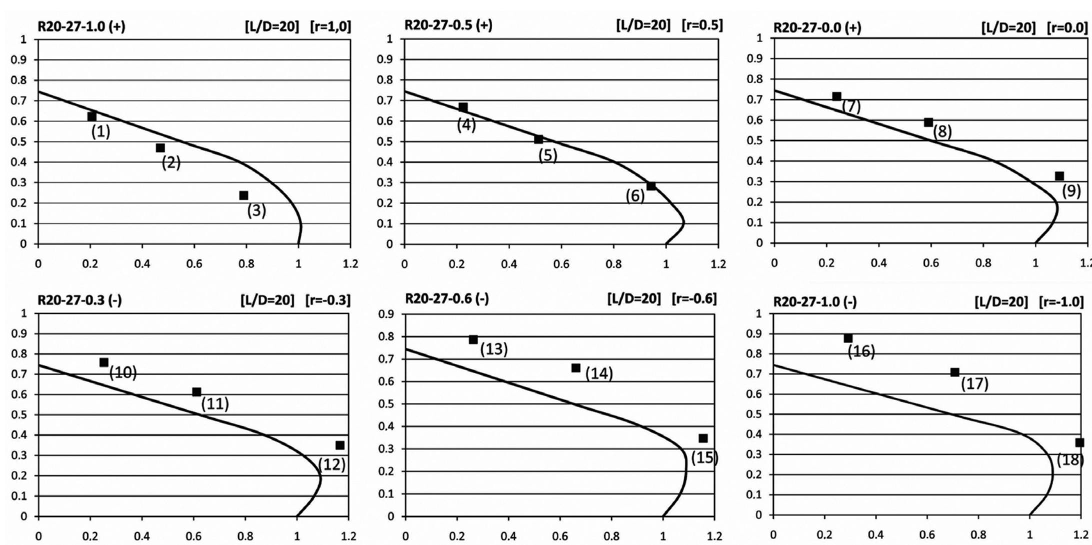

Figure 15. Comparison between experimental results obtained by [8] and the calculated curves, obtained through the proposed approach (case of slender columns).

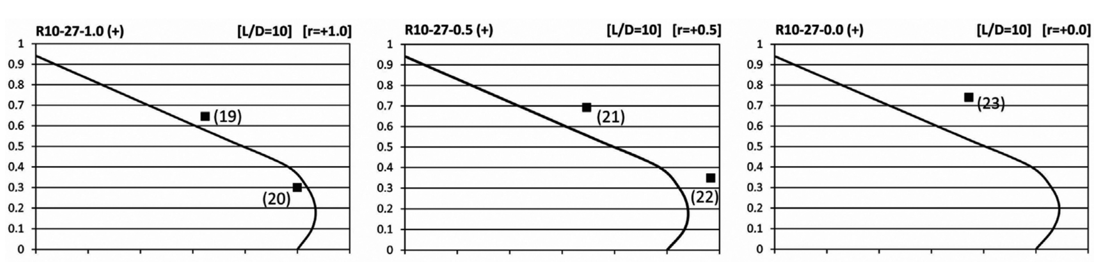

Figure 16. Comparison between experimental and calculated curves, obtained through the proposed approach (case of short columns).

eters of the section and ratio between end moments of the column. For low steel contribution ratios, the accuracy of the simplification is surprising.

It is important to point out that one of most important conclusions of this text is the election of determining variables in the behavior of a composite section. This way, the importance of the value δ and μ_{max} is shown through different chapters of this text: all mechanical parameters can be simply referred to the steel contribution ratio and curiously, the maximum moment ratio is independent of the shape of the section, as explained in Fig. 7.

This fact is worth of further research about the importance of this parameter on the behavior of concrete-filled tubes.

It is important to take into account that in case of a high shear ratio acting on the column, further analysis should be done in addition to the proposed methodology [9].

The proposed method presents substantial advantages in the design optimization process of this type of sections. This way, the text pretends to improve a verifying process that, complemented by existing sophisticated software, could cope effectively the determination of particular structural elements manually and at the same time, efficiently.

References

[1] N.E. Shanmugam, B. Lakshmi, State of the art report on steel-concrete composite columns, J. Construct. Steel Res. 57 (2001) 1041–1080.

[2] UNE-EN 1994-1-1:2011. EUROCODE 4. Design of composite steel and concrete structures Part 1.1: General rules and rules for buildings. European Committee for Standardization, Brussels, Belgium.

[3] M. Xinbo, S. Zhang, C.D. Goode, Comparison of design methods for circular concrete filled steel tube columns in different codes, in: Proc. of 8th International Conference on Steel-Concrete Composite and Hybrid Structures, Harbin, China, 12–14 August, 2006, pp. 30–37.

[4] ANSI/AISC-360-05. Specification for Structural Steel Buildings, American Institute of Steel Construction, Inc., Chicago, USA, 2005.

[5] Recommendations for Design and Construction of Concrete-filled Steel Tubular Structures, Architectural Institute of Japan (AIJ), Tokyo, Japan, 1997.

[6] S.J. Hicks, G.M. Newman, Design Guide for Concrete Filled Columns, Corus Tubes with Steel Construction Institute, Berkshire, UK, 2002.

[7] BS NA EN 1994-1-1: UK National Annex to Eurocode 4. Design of composite steel and concrete structures. General rules and rules for buildings. British Standards Institution (BSI).

[8] R. Bergmann, C. Matsui, D. Meinsma, D. Dutta, Guía de diseño para columnas de perfiles tubulares rellenos de hormigón bajo cargas estáticas y sísmicas', Comité International pour le Développement et l'Étude de la Construction Tubulaire, 1996.

[9] P. Moga, V. Pacurar, Circular composite columns. Mechanism of shear transfer, Ovid. Univ. Ann. Ser.: Civ. Eng. 8 (2006), November (1).