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Analytical Approach for Serviceability Assessment of Prestressed Concrete Girders Consisting of a Precast Beam and a Cast-in-place Slab

Enfoque analítico para la evaluación del Estado Límite de Servicio de vigas de hormigón pretensado que constan de una viga prefabricada y una losa hormigonada in situ

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ABSTRACT

This paper is focused on the assessment of the Serviceability Limit State of bridge girder cross-sections consisting of a precast, prestressed concrete beam and a cast-in-place top slab, which will be referred to as composite concrete sections because of the presence of more than one concrete within the section. Various methodologies for non-composite (i.e., with only one concrete within the section) concrete sections have been proposed by different authors to address the issue of stress assessment and crack control, mostly making use of analytical neutralization methods. In the present study, the neutralization method is extended and adapted to composite sections involving two concretes cast at different times. The proposed method is validated by comparison with a direct calculation method and commercial software. Three scenarios have been analyzed in the paper: midspan sections subjected to positive bending with cracking of the precast beam, sections at support regions of bridge girders subjected to negative bending in which the top slab is uncracked under permanent loads but undergoes cracking with the application of live loads, and sections at support regions in which the top slab is already cracked under permanent loads. Additionally, worked examples from real projects are provided.

KEYWORDS: prestressed concrete bridges, staged construction, cracking, composite sections, neutralization method.

RESUMEN

Este artículo se centra en la evaluación del Estado Límite de Servicio de secciones de puentes, consistentes en una viga prefabricada pretensada y una losa hormigonada in situ, que se denominarán secciones compuestas por la presencia de más de un hormigón en la sección. Diferentes autores han propuesto distintas metodologías para secciones homogéneas formadas por un solo hormigón, para abordar el cálculo de tensiones y el control de fisuras en su mayoría mediante el empleo de métodos analíticos basados en la neutralización. En el presente estudio, el método de neutralización se amplia y se adapta a secciones compuestas formadas por dos hormigones, hormigonados a diferentes edades. El método propuesto se valida mediante comparación con un método de cálculo directo y con el uso de un programa de cálculo comercial. Se han analizado tres escenarios: secciones de centro de vano sometidas a un momento flector positivo con fisuración de la viga prefabricada, secciones de apoyo sometidas a momento flector negativo donde la losa superior se fisura bajo la acción de la sobrecarga de uso, pero permanece comprimida bajo la acción de las cargas permanentes y secciones de apoyo donde la losa superior ya está fisurada bajo la acción de las cargas permanentes. Además, se añaden ejemplos de puentes de proyectos reales.

PALABRAS CLAVE: puentes de hormigón pretensado, construcción evolutiva o por fases, fisuración, secciones compuestas, método de neutralización.

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NOTATION

A Area of the cross section.
$A_{c,eff}$ Effective area of concrete in tension surrounding the reinforcement or prestressing tendons.
As_i Area of reinforcement steel of the layer i of reinforcement of the section.
Ap_i Area of prestressing steel of the layer i of prestressing of the section.
$B(x)$ First moment of inertia of the cracked transformed composite section at infinite time in relation to the fiber x of the cracked section.
$b(y)$ Width of the section.
$E_{cb}(t_\infty)$ Instantaneous Modulus of Elasticity of the concrete of the beam at end of time.
E_c Instantaneous Modulus of Elasticity of the concrete.
E_s Modulus of Elasticity of the reinforcement steel.
$I(x)$ Second moment of inertia of the cracked transformed composite section at infinite time in relation to the fiber x of the cracked section.
PN Neutralization Force.
c.o.g Center of gravity.
d_{pi} Distance from the layer "i" of prestressing steel to the top fiber of the section.
d_{si} Distance from the layer "i" of reinforcement steel to the top fiber of the section.
e Eccentricity of the neutralization force with respect to the center of gravity of the section.
e' Eccentricity of the neutralization force with respect to the center of gravity of the prestressing reinforcement of the beam.
e'' Eccentricity of the neutralization force with respect to the center of gravity of the prestressing reinforcement of the slab.
f_{ck} Compressive strength in the concrete.
$f_{ct,eff}$ Mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur.
k_1 Coefficient which takes into account of the bonded properties of the bonded reinforcement.
k_2 Coefficient which takes into account the distribution of strain.
k_3, k_4 Coefficients depending of National Annex.
k_t Factor dependent on the duration of the load.
n Relation between Modulus of Elasticity.
s_{rmax} Maximum crack spacing.
x Depth of neutral fiber of the cracked section.
y_g Position of the c.o.g referred to the bottom of the section.
$\Delta\varepsilon_{ci}$ Increase of strain in each fiber of concrete from state under permanent loads to zero stress plane in concrete.
ΔM Increment of bending moment due to live loads with respect to the permanent loads bending moment.
$\Delta\sigma_c$ Increase in stress in the concrete of the beam with respect to the concrete of the slab due to permanent loads at end time.
α_e Ratio between the Modulus of Elasticity of the reinforcement steel and the concrete.
ε_{sm} Mean strain in the reinforcement.
ε_{cm} Mean strain in the concrete between cracks.
ρ_{peff} Ratio between the area of bonded reinforcement and prestressing within Aceff.
σ_{bbct} Bottom beam concrete stress under permanent loads.
σ_{tbct} Top beam concrete stress under permanent loads.
σ_{bsct} Bottom slab concrete stress under permanent loads.
σ_{tsct} Top slab concrete stress under permanent loads.
σ_{ct} Concrete stress under permanent loads.
σ_c Concrete stress under permanent plus live loads.
σ_{pit} Prestressing steel stress under permanent loads.
σ_{sit} Reinforcement steel stress under permanent loads.
σ_{p0} Initial prestressing steel stress.
σ_{pi} Increase of stress in the prestressing steel due to live loads from zero stress plane in concrete.
σ_{si} Increase of stress in the reinforcing steel due to live loads from zero stress plane in concrete.
σ_{piN} Prestressing steel stress due to permanent loads at end time and neutralization forces.
σ_{siN} Reinforcement steel stress due to permanent loads at end time and neutralization forces.

1. INTRODUCTION

Many structural typologies of bridges rely on girder designs consisting of composite concrete sections (i.e., with more than one concrete within the section, typically manufactured at different times) affected by staged construction processes. Efficient structural systems may involve the utilization of precast, prestressed concrete beams and cast-in-place concrete top slabs, which are affected by time-dependent effects of the two concretes leading to progressive redistribution of stresses. The instantaneous effects produced by live loads might also induce cracking, which must be considered at the design stage [1][2] . Though composite sections consisting of a prestressed concrete beam and a reinforced concrete slab built at different times are mostly designed so that concrete stresses under the frequent load combination are lower than the concrete tensile strength to avoid the explicit calculation of crack widths, this assumption does not ensure that the sections remain uncracked under the actuation of eventual overloads or full live loads (i.e., the characteristic combination of loads). In the previous situation, the section might come back to fully compressed state after removal of live loads, but the previous decompression cancels the further ability to carry tensile stresses under new load increments including the frequent load combination. Moreover, partially prestressed composite sections also require crack width estimation in the serviceability verifications. Accordingly, consistent analytical methods to assess stress state and crack control of composite concrete sections are necessary.

Serviceability Limit State (SLS hereafter) of cracking in composite sections consisting of a precast, prestressed concrete beam and a cast-in-place top slab was discussed in the first author's PhD Thesis [3]as a complement to the main aim of the thesis, which was to develop a simplified method for time-dependent effects in statically indeterminate concrete bridges with connected precast beams with evolutive cross-sections, also detailed in [4]. The necessity of analytical models was discussed and highlighted. The problem of a non-composite (i.e. with only one concrete within the section) prestressed concrete section which is fully compressed under permanent loads but undergoes cracking with the additional application of live loads -partially prestressed concrete section- has been addressed by several authors. Nilson [5] and Dilger *et al.* [6] used approaches based on the neutralization of prestressing stresses. Ghali [7] proposed the neutralization of the stresses produced by permanent loads. Karayannis *et al.* [8] proposed a simplified method mainly for T-sections and completed a parametric study focused on the amount of prestressing to study the crack width. Lee *et al.* [9] suggested a simplified method for the calculation of stress increase in passive reinforcement and performed a parametric study on T and inverted T sections to check that tensile stress of prestressing strands is under 250 MPa due to ACI 318 code [2] formulation.

Cracking of composite sections has been addressed by various authors. Amongst others, Naaman *et al.* [10] firstly solved the composite section without considering time-dependent effects (shrinkage, creep and relaxation) or the variation over time of the concrete modulus of elasticity. In subsequent research, Naaman *et al.* [11] updated the direct method for instantaneous actions with the use of elastic deformations for equilibrium conditions and total deformations (instantaneous and delayed) for compatibility conditions. Despite of the quality of the paper, three drawbacks can be highlighted: compatibility conditions should be extended to total curvatures (elastic and time-dependent), the instantaneous modulus of elasticity of concrete was assumed constant for all periods (neglecting the initial value at time of prestressing, notably lower), and the fact of combining as a single stage the delayed effect of casting the top slab, the application of superimposed loads and the instantaneous effect of the live load that causes cracking, which are clearly different phenomena. Pokharel [12] relied on the general recommendations and assumptions from Naaman *et al.* [11] however without providing details of the calculation steps for the analysis of cracking of the composite section.

The most extended analytical solution so far is based on the neutralization method in order to overcome the inconsistency of applying superposition to load sequences governed by the nonlinearity of concrete cracking: in a first stage, the permanent stress state of the concrete is cancelled with the application of a couple of neutralization sectional forces (an axial force and a bending moment, i.e. an eccentric neutralization axial force); in a subsequent second stage, neutralization forces are applied in opposite direction together with the sectional forces produced by live loads to a composite section in which the concrete does not carry tensile stresses (cracked section analysis). The extension of the neutralization method to composite sections including more than one concrete (typical of bridge girders consisting of a precast beam and a cast-in-place slab, manufactured at different times) is not straightforward. Even though Ghali *et al.* [13] pointed out that the problem can be solved with neutralization forces by cancelling the stresses of the concrete which cracks, the subsequent calculation steps were not described, and no practical examples were provided. In practice, commercial software for the calculation of cracking in prestressed sections formed by one or several layers of concrete are available [14]-[19] but the accuracy of the analysis will depend on the correct definition of assumptions, mechanical properties of materials and loading conditions within the software. Although some software tools do not have a specific function for cracking assessment, detailed structural analysis can be performed to evaluate stresses and strains in composite sections and the occurrence of cracking can be inferred from the calculated stress and strain distribution; however, the introduction of the previous creep and shrinkage of concrete layers is not always obvious.

In the present paper, the neutralization method has been extended and adapted to composite sections consisting of a prestressed concrete beam and a cast-in-place concrete slab [3], i.e. with two concretes cast at different times. A summary of the neutralization method for prestressed concrete sections with one concrete is presented in Section 2 based on [20]. In Section 3, the application to the most relevant cases of bridge girders with two concretes within the section is detailed: mid-span region sections subjected to positive bending moment (cracking of the precast beam), support region sections subjected to negative bending moment in which the top concrete slab is uncracked under permanent loads but cracks due to the application of live loads, and support region sections where the top slab is already cracked under permanent loads. Section 4 is dedicated to validating the proposed method for composite sections. This involves comparing results with direct calculations and commercial software [14]. Section 5 encompasses practical examples derived from real projects. In Section 6, the attention is paid at the influence of the modulus of elasticity of the concrete which can be used for the cracked section analysis (either instantaneous or delayed). Lastly, conclusions are drawn in Section 7 of the paper.

2.
BACKGROUND TO THE NEUTRALIZATION METHOD FOR PRESTRESSED CONCRETE SECTIONS

The stress state development of a prestressed concrete section with one concrete and two types of embedded steel layers (reinforcing and prestressing, both perfectly bonded to the concrete), subjected to the subsequent action of permanent and live loads is a common problem in the structural analysis of prestressed concrete girders for serviceability verification. According to design codes, stress state assessment and crack control must be verified for different fractions of the live loads: quasi-permanent, frequent and/or characteristic load combinations [1]. Typically, the section is fully compressed under permanent loads but undergoes cracking at the most tensioned side with the actuation of a fraction of the live loads. The application of the superposition method to add the stresses caused by permanent and live loads is not consistent due to the nonlinearity which arises when concrete cracks. As an alternative to numerical non-linear approaches, the problem can be solved analytically in two stages with the help of the neutralization method [20]. In Figure 1, the stress

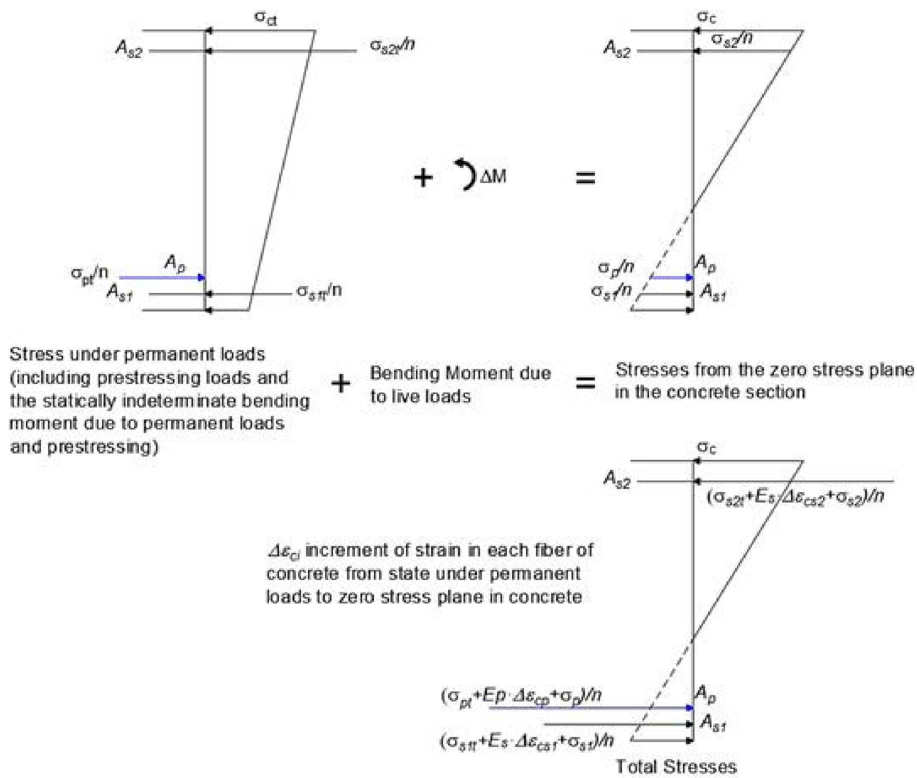


Figure 1. Stress development in a prestressed concrete section from the permanent stress state after the application of live load.

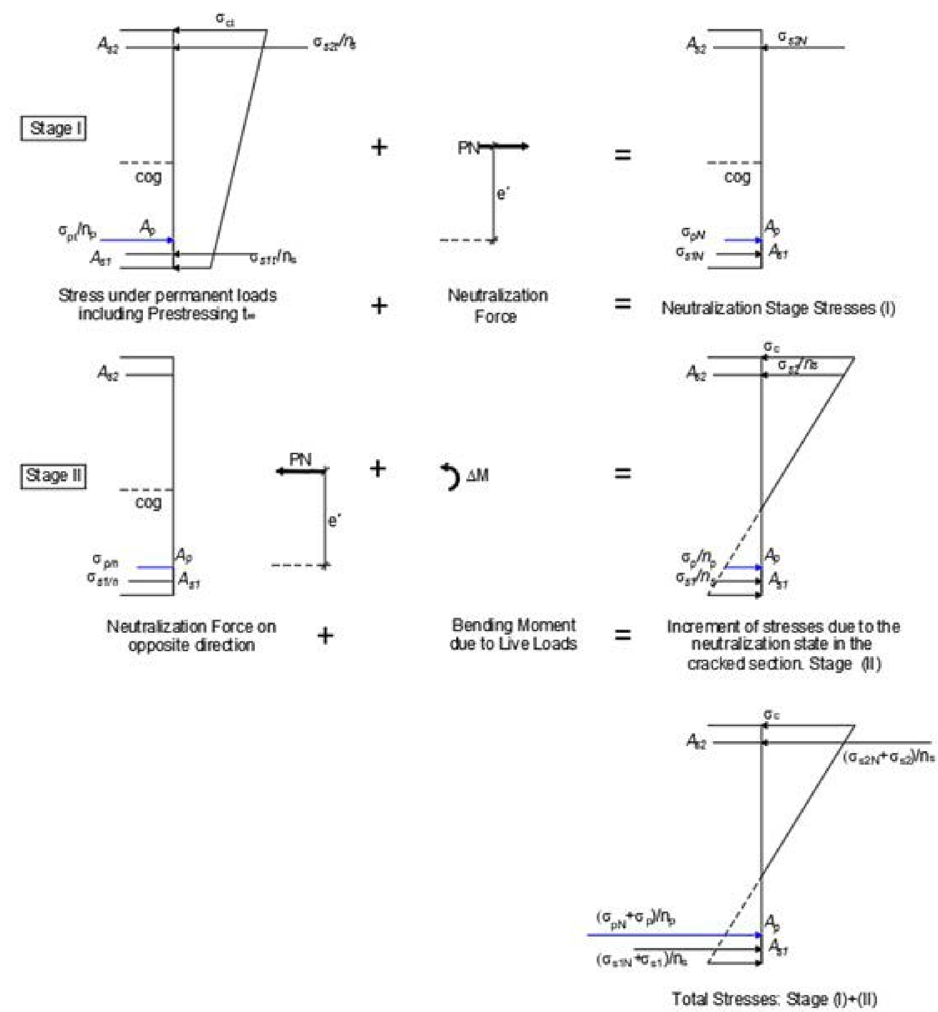


Figure 2. Neutralization method. Decomposition of stresses.

development of the prestressed concrete section is shown. The change of stresses from the uncracked permanent state to the cracked situation under live loads can be addressed with the neutralization method, by the decomposition of the process in two steps (Figure 2.

In the first step (Stage I in Figure 2), the neutralization force PN, applied with an eccentricity e’ with respect to the centroid of the prestressing steel, is obtained by cancelling the permanent concrete stresses with use of the transformed section properties: the total area of the section is a result of the concrete area plus the area of each steel layer (Asi and Api) multiplied by the corresponding modular ratio with respect to concrete (As_i·Es/Ec and Ap_i·Ep/Ec respectively). The resulting concrete stresses are zero in the so-called neutralization plane, but not zero in the prestressing and non-prestressing steels, so that they balance the system of forces formed by the permanent loads plus the neutralization force.

In the second step (Stage II in Figure 2), the neutralization force is applied in opposite direction plus the bending moment caused by live loads from the neutralization plane. The solution to Stage II can be obtained with the traditional assumptions of the cracked transformed section: plane sections remain plane, linear elastic behaviour of the steels and the concrete in compression, and concrete cannot carry tensile stresses.

The basis of the method is to cancel concrete stresses in the neutralization state, which allows the stresses of Stages I and II for steel and concrete to be superimposed, since the non-linearity of concrete (which would prevent the addition) is cancelled as the first term of the sum is zero and cracking appears automatically (zero-stress layers) in Stage II. In the case of concrete, as the neutralization stresses from Stage I are zero, the stress increments of Stage II coincide with the final stresses and directly include the non-linear effect of cracking through the zero-stress zone at the bottom of the section. For prestressing and non-prestressing steels, final stresses are the sum of those corresponding to the neutralization Stage I plus those of Stage II. This sum is possible because in service the steels behave elastically without yielding. Stage II can be easily solved as a concrete section with several layers of reinforcement subjected to an axial force and a bending moment (Figure 3).

According to the classical theory, applying sectional equilibrium of axial force and bending moment with respect to the point of zero stresses, referred to as 0 in Figure 3:

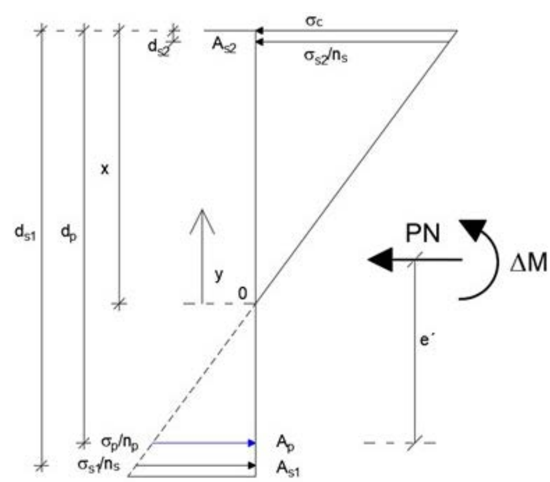


Figure 3. Cracked section analysis from the neutralization state.

$$PN=\frac{\sigma_c}{x}\int_0^x b(y)y\,dy+n\frac{\sigma_c}{x}A_{s2}\,(x-d_{s2})-n\frac{\sigma_c}{x}A_{s1}\,(d_{s1}-x)-n\frac{\sigma_c}{x}A_p\,(d_p-x) \quad (1)$$

$$\Delta M-PN(d_p-e'-x) = \frac{\sigma_c}{x}\int_0^x b(y)y^2dy+n\frac{\sigma_c}{x}A_{s2}\,(x-d_{s2})^2-n\frac{\sigma_c}{x}A_{s1}(d_{s1}-x)^2 -n\frac{\sigma_c}{x}A_p\,(d_p-x)^2 \quad (2)$$

Equations (1) and (2) can be written in a simplified form:

$$PN=\mathrm{B}(x)\,\frac{\sigma_c}{x} \quad (3)$$

$$\Delta M- PN(d_p-e'-x)=I(x)\,\frac{\sigma_c}{x} \quad (4)$$

where *B(x)* and *I(x)*, are the first and second moment of inertia of the transformed cracked section with respect to the neutral axis. The former system of equations is general for any section shape with a vertical axis of symmetry and several layers of prestressing and reinforcing steel. It allows obtaining the neutral axis depth (x) and the top concrete stress (σ_c). Hence, by compatibility, the complete distribution of stresses at the section can be derived. It must be noted that the transformed section in Stage I is homogenized to the instantaneous modulus of elasticity of concrete because first cracking takes place due to the action of the frequent or the characteristic combination of loads, which is an instantaneous phenomenon. Moreover, the same modulus of elasticity must be used in the two stages I and II for the sake of consistency in the addition.

3. EXTENSION OF THE NEUTRALIZATION METHOD TO COMPOSITE SECTIONS WITH MORE THAN ONE CONCRETE

In this Section, the neutralization method is extended to composite sections including more than one concrete, which is useful for prestressed concrete girders constructed sequentially. In particular, the attention is paid at the SLS verification of composite sections consisting of a precast, prestressed concrete beam and a reinforced concrete cast-in-place top slab. In practice, there are methods [5][6] to neutralize prestressing stresses which are valid for prestressed sections with only one type of concrete, but they are not generally correct for composite sections. In particular, the attention is paid at sections consisting of a precast, prestressed concrete beam and a top cast-in-place concrete slab. Three different cases are addressed, including sections subjected to positive and negative bending moments:

- Case A: sections at midspan regions of bridge girders subjected to positive bending with cracking of the precast beam.
- Case B: sections at support regions of bridge girders subjected to negative bending in which the top slab is uncracked under permanent loads but undergoes cracking with the application of live loads.
- Case C: sections at support regions of bridge girders subjected to negative bending in which the top slab is already cracked under permanent loads.

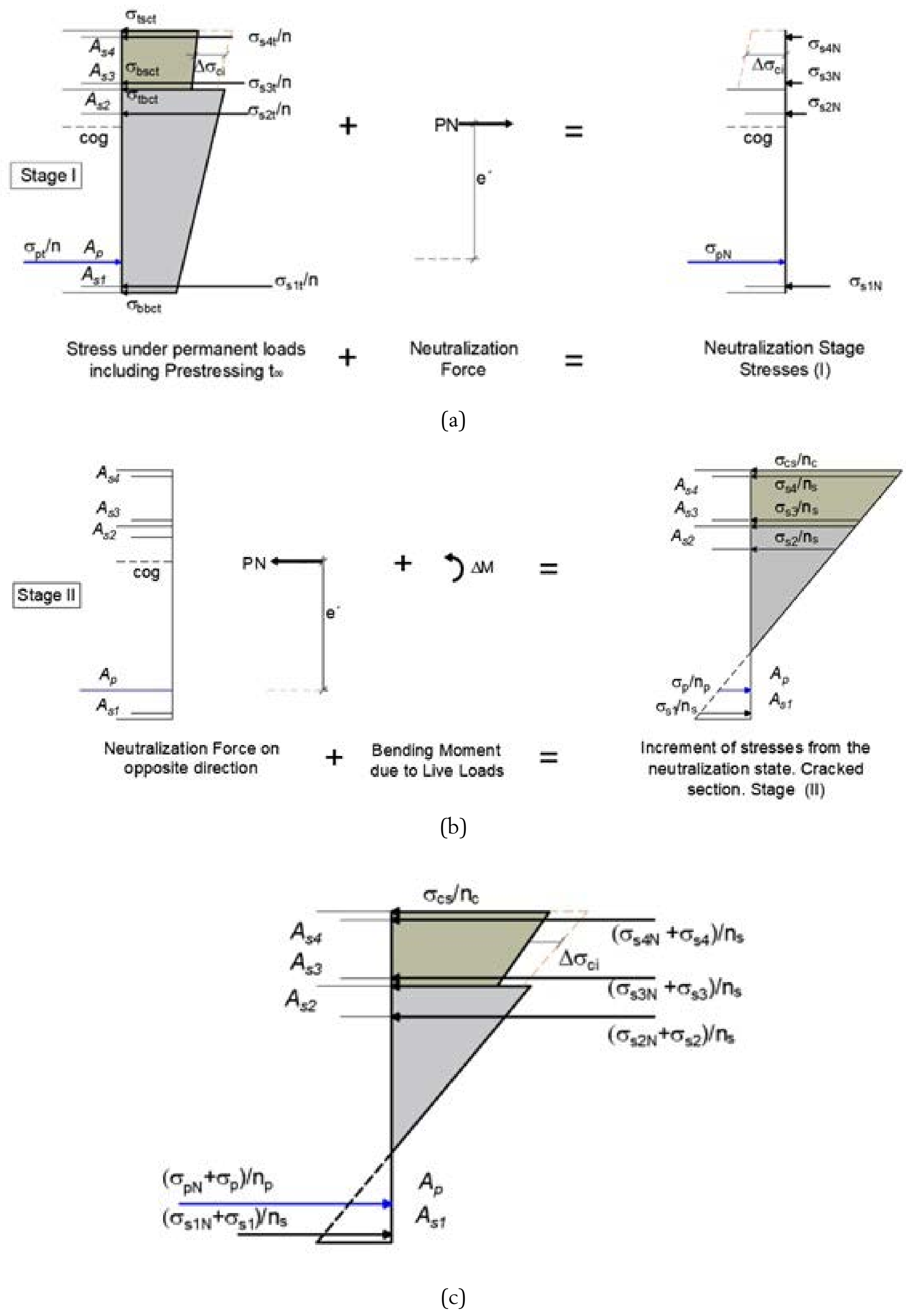


Figure 4. Composite section at midspan regions subjected to positive bending moment with cracking of the precast beam. (a) Stage I: Neutralization of permanent concrete stresses at the precast beam. (b) Stage II: Increment of stresses from neutralization state due to application of neutralization forces in opposite direction and live loads. (c) Total Stresses: Stages I + II.

3.1. *Case A*

In this case, the actuation of positive (sagging) bending moment at midspan regions of bridge girders decisive for cracking at the bottom of the precast prestressed beam under live loads is studied, whereas the top slab remains in compression under all combinations of loads (quasi-permanent, frequent, and characteristic). The application of the neutralization method to obtain the stress change due to the application of live loads can be explained with the help of Figure 4.

Under these conditions, the permanent stress state at end time is a result of permanent loads, including prestressing forces, and time-dependent effects of the two concretes and their interaction [3][4]. Concrete stresses at the top and bottom of the slab are referred to as σ_{tsct} and σ_{bsct}, concrete stresses at the top and bottom of the precast beam are σ_{tbct} and σ_{bbct}, and the non-prestressing and prestressing steel stresses are σ_{sit} and σ_{pt} respectively. From such permanent state, Figure 4a summarizes the process of stress neutralization (Stage I). The neutralization force (PN) and its eccentricity (*e'*) with respect to the centroid of the prestressing layer are obtained in the transformed composite section by cancelling the permanent concrete stresses of the precast beam.

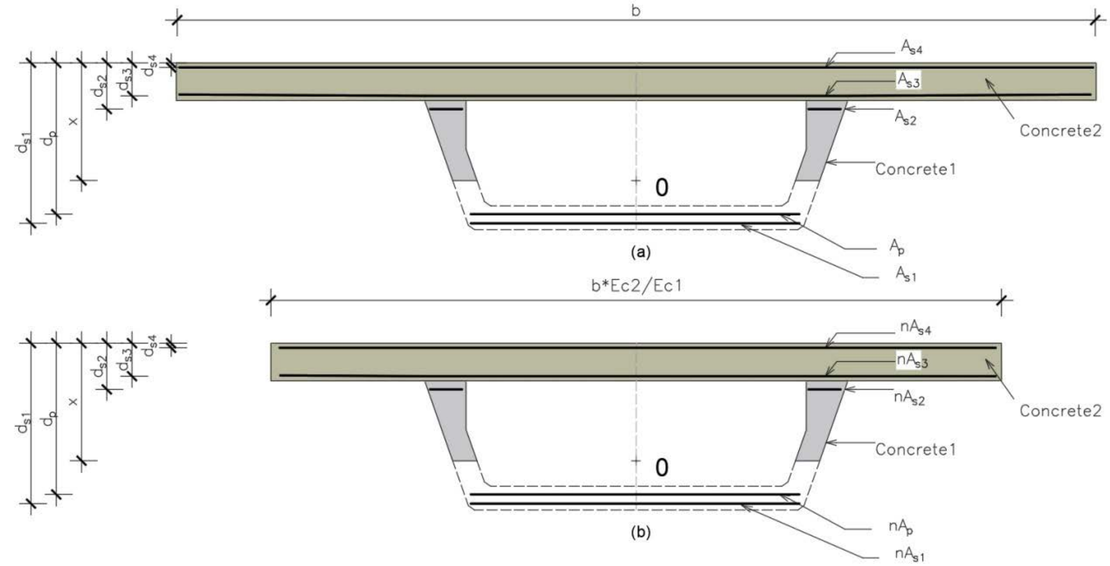


Figure 5. (a) Cracked section (b) Transformed cracked section.

The stresses in prestressing and reinforcing steels obtained in the neutralized state are σ_{piN} and σ_{siN} respectively. Non-zero stresses also result in the concrete of the top slab in the situation of neutralization of stresses. The value of $\Delta\sigma_{ci}$ in Figure 4a is the difference between the concrete stresses at the top slab and those at the precast beam multiplied by their modular ratio. This value is constant for each fiber of the concrete of the slab throughout the process as the process assumes different strain planes (the difference of concrete stresses of the precast beam multiplied by the modular ratio is shown by the dashed orange line in the Figure 4a). In Figure 4b, Stage II of the method is schematized: the neutralization force is applied in opposite direction in addition to the bending moment due to live loads. The properties of the section are those corresponding to a transformed section where the concrete slab is homogenized with its modular ratio with respect to the concrete of the precast beam, (Figure 5).

This stage provides the stress increase from the neutralization plane. At this stage, cracking of the concrete beam occurs. The transformed cracked section method explained in Section 2 for Stage II can be applied, as mentioned above, with the top concrete slab playing a similar role as other layer of prestressing steel (the analogy makes sense, as the initial strain of the prestressing steel, would be analogous to the strain difference which initially exists between the concrete of the top slab and that of the precast beam due to the effect of the self-weight of the top slab on the beam while the concrete of the top slab hardens, as well as other initial strains of the precast beam). Figure 6 shows the stresses in the cracked transformed section in Stage II. The calculation of the neutral axis depth, (x in Figure 6) is carried out by applying the equilibrium, Equations (3) and (4). In this case, $B(x)$ and $I(x)$ are the first and second moment of inertia of the cracked transformed section with respect to the neutral axis and sc is the concrete stress at the top slab divided by n_c –ratio of E_i of both concretes–, and d_p and x are measured from the top of the slab.

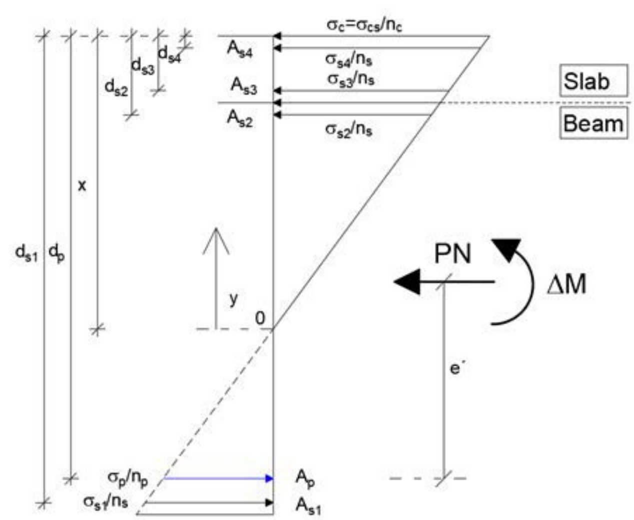


Figure 6. Stage II for Case A. Cracked section analysis.

The final total stresses (Figure 4c) are those of Stage II for the concrete of the precast beam (zero stresses in the neutralization process), while the stresses for the concrete of the top slab, the reinforcing and prestressing steels are the sum of the stresses of Stages I and II. The method is valid because the reinforcing and prestressing steels behave linear-elastically, as well as the concrete of the top slab as long as it is always compressed.

3.2. *Case B*

A composite section subjected to negative bending in which the top slab is uncracked under permanent loads is studied here. Such a situation might occur when the top slab is prestressed, though such sections are usually oversized, and cracking does not occur normally.

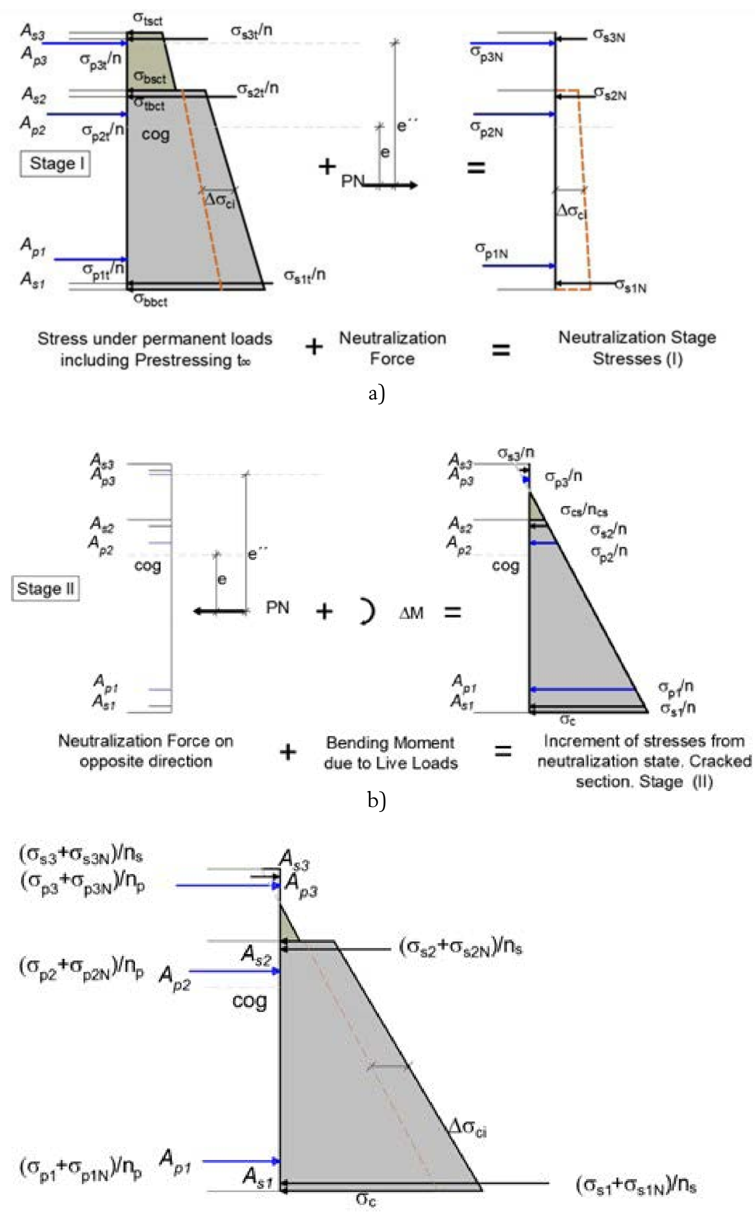


Figure 7. Composite section at support regions girders subjected to negative bending moment in which the top slab is uncracked under permanent loads but undergoes cracking with the application of live loads. (a) Stage I: Neutralization of permanent concrete stresses at the top slab (b) Stage II: Increment of stresses from neutralization state due to application of neutralization forces in opposite direction and live loads (c) Total Stresses: Stages I + II.

The solution thus is based on the procedure explained in Section 3.1 with the top prestressed concrete slab playing the same role as the prestressed beam in Case A. Stage I consists of the introduction of an eccentric neutralization force PN to cancel the permanent concrete stresses of the top slab Figure 7a.

Stage II (Figure 7b) corresponds to the application of the neutralization force in opposite direction to the cracked transformed section plus the bending moment due to live loads. Concrete stresses at the top slab after the neutralization are zero, while the stress increase of the concrete of the precast beam with respect to the concrete of the top slab multiplied by the modular ratio between both concretes ($\Delta\sigma_{ci}$) remains constant throughout the process in each fiber of the concrete of the beam. It is necessary to verify that concrete stresses at the precast beam remain in compression during the whole process. Cracking of the top slab occurs in Stage II. Total stresses due to permanent and live loads are obtained by adding those from the Stages I and II (Figure 7c).

In this case, the cracked section is solved by applying the scheme shown in Figure 8. The system of equations is the same as the one defined by Eqs. (3)-(4) but replacing d_p and e´ with d_{p3} (distance from the prestressing steel of the top slab to the bottom of the precast beam) and $e´´$ (eccentricity of the neutralization force with respect to the prestressing steel of the top slab), respectively.

It might occur that the neutral axis depth goes into the beam in Stage II. In such a situation, the concrete of the beam between the neutralization Stage I to Stage II remains compressed. This results in a decrease of compressive stresses in the beam (Figure 9), which is represented by some tensile

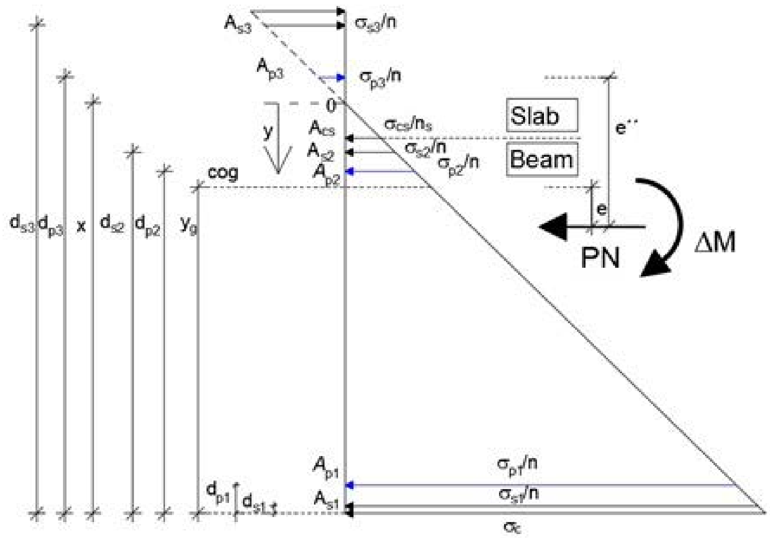


Figure 8. Cracked section analysis from the neutralization state for a section subjected to negative bending moment.

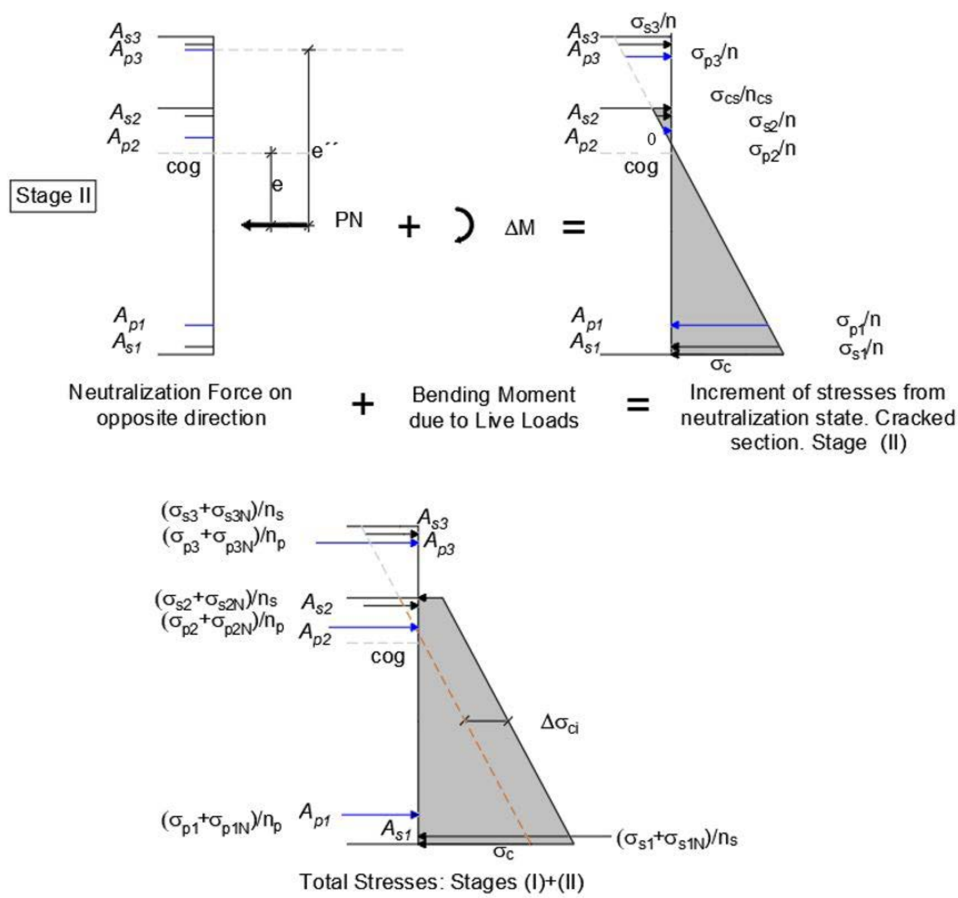


Figure 9. Figure 7b and 7c modified with neutral fiber x inside the depth of the precast beam.

stresses in the concrete of the upper zone of the beam up to the neutral axis (Stage II is an incremental process starting from the plane of neutralization of the stresses in the slab). Figure 9 shows Stage II and the total stresses sum of Stages I (the same as Figure 7a) and II.

The mechanical effect of the tensile stresses of the concrete of the beam can be introduced in the calculations by including it in the equilibrium conditions in Stage II. Equations (3) and (4) can be used, applying sectional equilibrium of axial force and bending moment with respect to the point of zero stresses, referred to as 0, see Figure 9. In addition, the expressions for *B(x)* and *I(x)*, the first and second moment of inertia of the cracked transformed section with respect to the neutral axis, must include the contribution of the complete area of the beam.

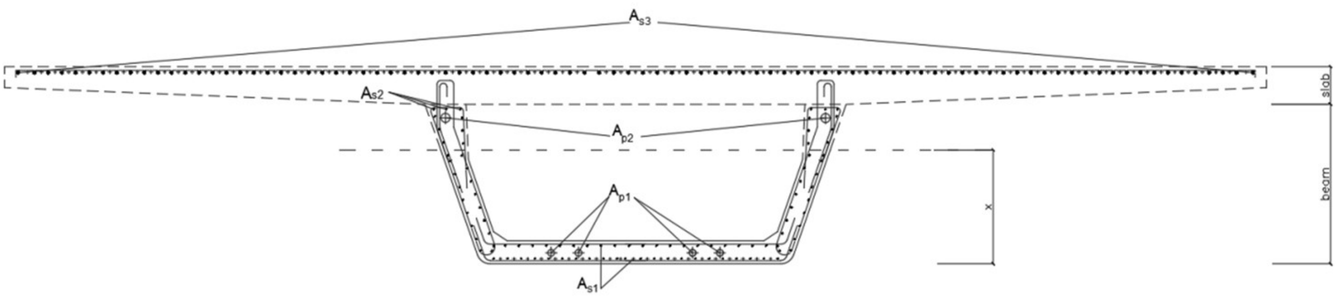
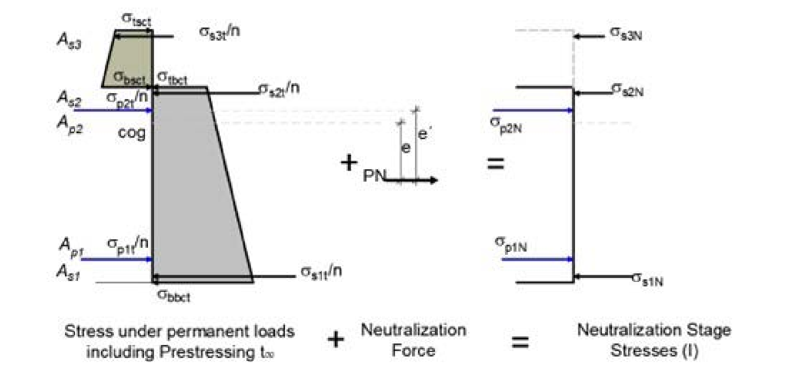
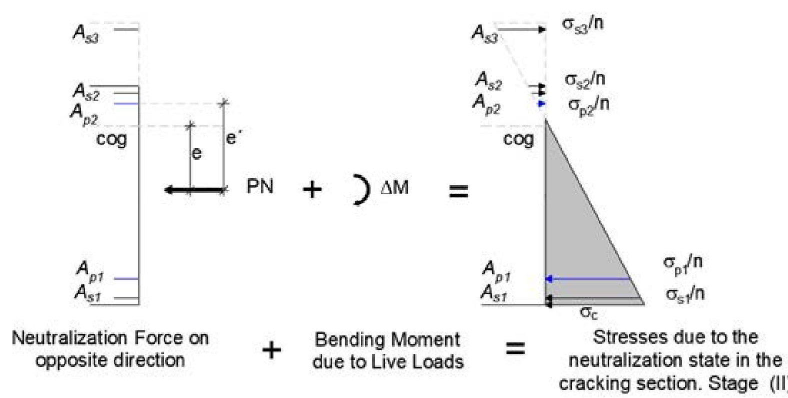


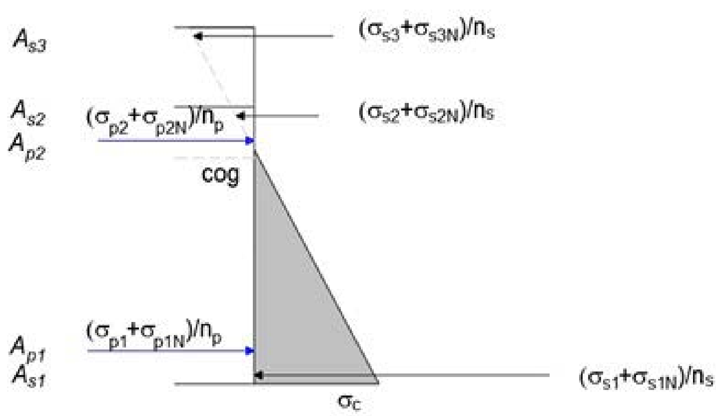
Figure 10. Cracked Composite Section. Case C.



(a)



(b)



(c)

Figure 11: Composite sections at support regions subjected to negative bending in which the top slab is cracked under permanent loads. (a) Stage I: Neutralization of the permanent concrete stresses at the precast beam. (b) Stage II: Increment of stresses from neutralization state due to application of neutralization forces in opposite direction and live loads. (c) Total Stresses: Stages I + II.

3.3. *Case C*

This Section analyzes the case of a composite section consisting of a precast prestressed beam and a cast-in-place top slab subjected to negative bending moment, in which the top slab is cracked under permanent loads at infinite time.

In this case the concrete of the top slab is considered ineffective, so the considered section is a prestressed concrete section with only one concrete and the reinforcing steel of the top slab outside the concrete (Figure 10).

The application of the method consists, in Stage I, of the neutralization of the permanent concrete stresses in the precast beam (including prestressing loads) at end time (Figure 11a). Stage II (Figure 11b) corresponds to the application of the neutralization force in opposite direction plus the bending moment due to live loads to the cracked transformed section. Total stresses due to permanent and live loads are obtained by adding those of Stages I and II (Figure 11c).

The cracked section is solved by applying the scheme shown in Figure 12 and the system of equations is the same as the one defined in Eqs. (3) and (4) but referring d_p and e' to the prestressing steel of the precast beam (A_{p2}).

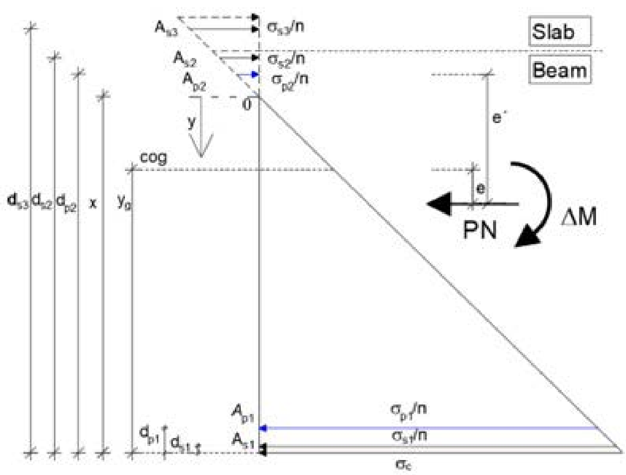


Figure 12: Cracked section analysis from the neutralization state for a section subjected to negative bending moment in which the top slab is cracked under permanent loads.

4. VALIDATION

In this Section, the capabilities of the proposed approach based on the neutralization method are verified by a comparison of the results with a direct calculation and the commercial software FAGUS [14]. FAGUS is part of CUBUS Engineering Software which allows for the cross-sectional analysis of structural elements and gives the stress distribution for a given combination of sectional forces. A reference example is studied to validate the neutralization method for composite sections.

4.1. *Reference example*

An example is studied of a composite section consisting of a rectangular pre-tensioned concrete beam of 1.0 m (height) x 0.3 m (width) and a top concrete slab of 0.25 m (thickness) x 0.3 m (width), refer to Figure 13. The composite section is supposed to be at the midspan region of a structural element subjected to positive bending which is compressed under permanent loads but cracks with the application of live loads. The initial stress at the prestressing steel is σ_{p0} = 1400MPa, the area of prestressing steel A_p is 3.46 cm² and is located 5 cm from the bottom of the beam. For simplicity, both components of the cross-section (beam and slab) have the same concrete and the modular ratio of the prestressing steel (Ep/Ec) is 6.0. Reinforcing steel and rheological effects of concrete are not considered for the sake of simplicity of the example.

The relevant mechanical properties (area, second moment of inertia and distance of the centroid to the bottom of the transformed section) of the beam section and the composite section, both homogenized to concrete as reference material, are as follows:

Transformed beam section:
A = 0.30173 m², I = 0.02534 m^4, y_g = 0.4974 m.

Transformed composite section:
A = 0.37673 m², I = 0.04939 m^4, y_g = 0.6223 m.

The beam is subjected to positive bending moments due to its self-weight and the weight of the slab of 210 kN·m and 53 kN·m, respectively. The composite section is subjected to a bending moment due to live loads of 190 kN·m. As there are no shrinkage and creep effects, there is no stress redistribution and, therefore, the slab has no stresses before the application of live loads.

4.2. *Direct calculation*

The stresses in the beam due to prestressing and its self-weight are -1.469 and -1.745 MPa at the top and at the bottom of the section, respectively, which can be easily obtained from Navier's equations with the use of the mechanical properties of the transformed beam section because the beam section is fully compressed.

The additional stresses caused by the self-weight of the cast in place slab modify the stresses of the beam so that the total stresses due to permanent loads are -2.520 and -0.704 MPa at the top and at the bottom of the beam, respectively (Figure 13a, note that the self-weight of the slab is totally carried by the beam because the fresh concrete of the slab does not contribute yet). Once the top slab has hardened, the bending moment due to live loads is applied to the composite section. Since such additional bending moment leads to cracking of the beam, the superposition principle cannot be applied to sum the stresses due to dead loads and live loads, as shown in Figure 13. Direct application is possible since concrete creep and shrinkage are not included in the example. For the application of a direct calculation, the following assumptions have been made:

- The plane of zero stresses coincides with the plane of zero strain. Therefore, the pre-tensioned steel stress is 1400 MPa at the zero-stress plane.
- As plane sections remain plane, the compression resultant at the slab (C), which is the continuation of the beam

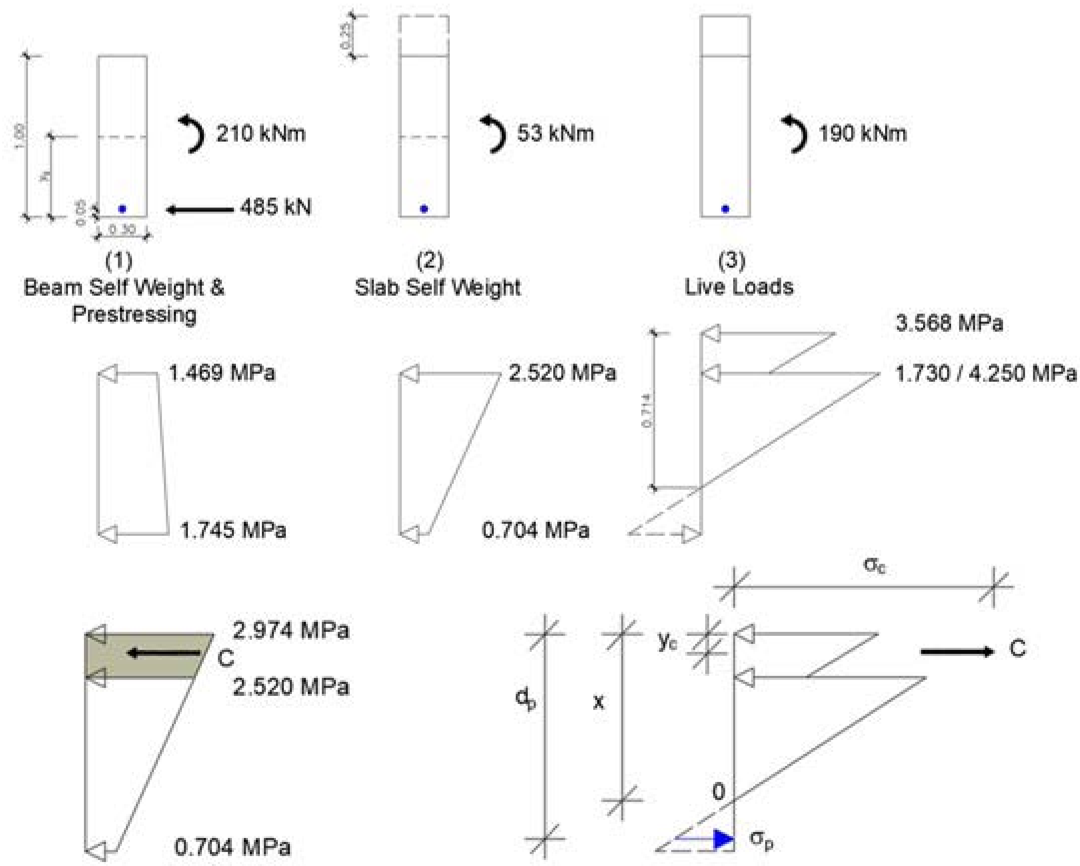


Figure 13. Validation example. Stresses direct calculation.

stresses at permanent loads are discounted in the calculation of final stresses.

The problem is solved by applying sectional equilibrium of axial force and bending moment with respect to the point of zero stresses, referred to as 0 in Figure 13. The axial force in the example is zero and the bending moment (M) is the sum of the bending moment due to dead loads and live loads:

$$0 = \frac{\sigma_c \cdot x \cdot b}{2} - C - \left(P + n\frac{\sigma_c}{x} A_p (d_p - x) \right) \tag{5}$$

$$M = \frac{\sigma_c \cdot x^2 \cdot b}{6} - C(x - y_c) + \left(P_p + n\frac{\sigma_c}{x} A_p (d_p - x) \right)(d_p - x) \tag{6}$$

where b is width of the cross section, C is the stress resultant at the slab from continuation of the beam stresses at permanent loads (Figure 13), yc is the distance from the centroid of the stress resultant C to the top of the slab, and σ_c is the stress at the top of the slab considering the value of C, Figure 13.

Solving Eqs. (5) and (6) yields a neutral axis depth of $x = 0.7136$ m and a compressive stress at the top of $\sigma_c = 6.542$ MPa. Therefore, the stresses at the top and bottom of the slab, and the top of the beam, are -3.568, -1.730 MPa and -4.250 MPa, respectively.

4.3. *Neutralization method*

Stage I of the neutralization method consists of the calculation of the neutralization force PN and its eccentricity e' with respect to the prestressing steel layer, so that the permanent concrete stresses in the beam are cancelled (-2.520 MPa and -0.704 MPa at the top and the bottom of the beam, respectively) with the use of the transformed composite section properties. The resulting neutralization force is PN = 691.02 kN with an eccentricity e´ = 0.702 m. With the neutralization forces, the resulting stresses at the top and the bottom of the slab are tensile stresses 2.974 MPa and 2.520 MPa, respectively (see Figure 14).

In Stage II, the neutralization force is applied in opposite direction in addition to the bending moment due to live loads. With the equilibrium equations (3) and (4), the resulting neutral axis depth and concrete compressive stress at the top of the composite section (x and σ_c in Figure 6) are 0.7082 m and 6.594 MPa, respectively. Therefore, the total stresses at the top and the bottom of the slab, and at the top of the beam are -3.620, -1.746 MPa and -4.266 MPa, respectively. The neutralization method with its two stages is depicted in Figure 14.

4.4. *Commercial software program FAGUS*

FAGUS is a program for the definition and design of cross-sections used in the general CUBUS software framework [14]. FAGUS analysis method enables the calculation of stresses based on specified strains or forces. In the first scenario, known the strain plane allows for a straightforward integration of stresses over the cross-section to determine the section forces. Assumptions include that cross-sections remain plane (Bernoulli hypothesis), complete bonding between concrete and steel, and zero concrete tensile strength (implying a cracked

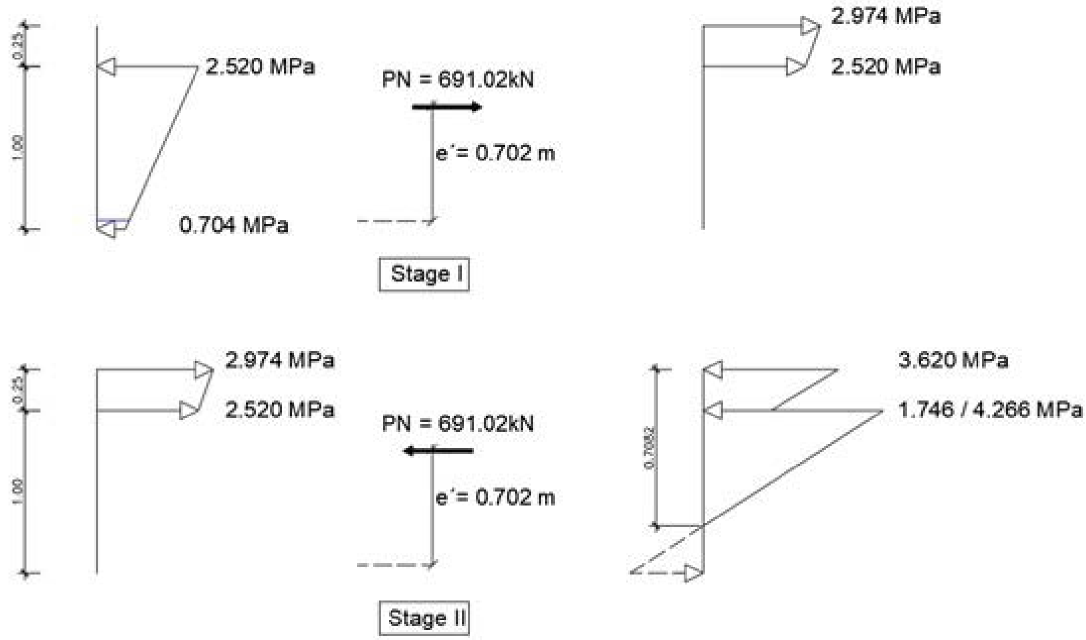


Figure 14. Validation example. Neutralization method.

Extreme stresses and strain after 5 steps

Name	Class	y_q [m]	z_q [m]	ε [‰]	σ_d [N/mm²]	γ [-]
C1	C50/60	0.3	1.	-0.1	-4.276	1.00
C1	C50/60	0.	0.	0.1	0.	1.00
PP1	S1670/1860	0.15	0.05	7.3	1424.815	1.00

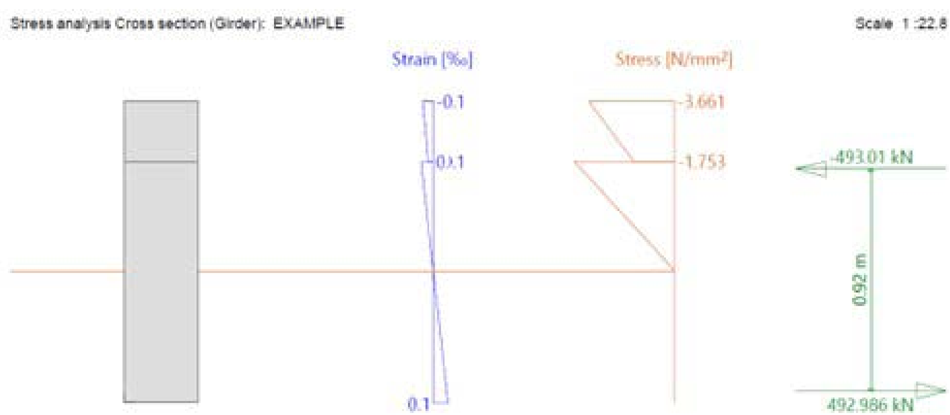


Figure 15. Validation example. Calculation with Fagus.

concrete tensile zone). In the second scenario, analyzing stresses for given forces demands additional computational effort. Determining the associated strain plane an iterative process which involves assuming a strain plane, calculating internal forces and moments (cross-section integration), and then comparing external and internal forces. If discrepancies are significant, the iteration is repeated with an improved strain plane.

FAGUS can conduct stress checks for any composite cross-section type via batch analysis, gradually introducing loads or deformations on selective partial sections that can be activated or deactivated at each step.

In the reference example, the stresses at the top and at the bottom of the slab resulting from the analysis with FAGUS are -3.661 and -1.753 MPa, respectively. The stress at the top of the beam is -4.276 MPa, as shown in Figure 15. Therefore, the neutral axis depth x is equal to 0.7021 m.

TABLE 1.
Validation. Example

	Direct Calculation	Direct Calculation/ FAGUS	Neutralization Method	Neutralization Method/ FAGUS	FAGUS
X(m)	0.7136	1.017	0.7082	1.009	0.7020
σ_{tsc} (MPa)	3.568	0.975	3.620	0.989	3.661
σ_{bsc} (MPa)	1.730	0.987	1.746	0.996	1.753
σ_{tbc} (MPa)	4.250	0.994	4.266	0.998	4.276

The results are shown in Table 1, which compares the neutral axis depth and the concrete stresses at the top and bottom of the slab, and at the top of the beam, with the three methods studied: direct calculation, neutralization method, and FAGUS software. Both direct calculation and neutralization method give

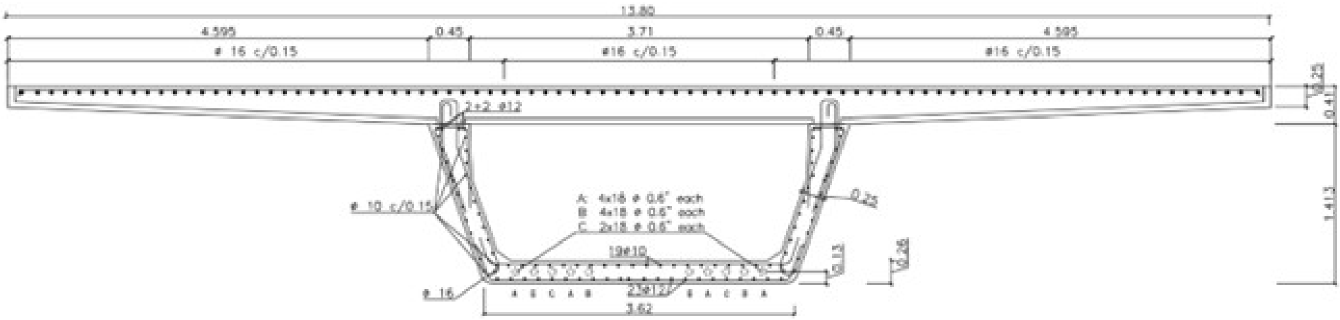


Figure 16. Cross section at midspan. Section 1.

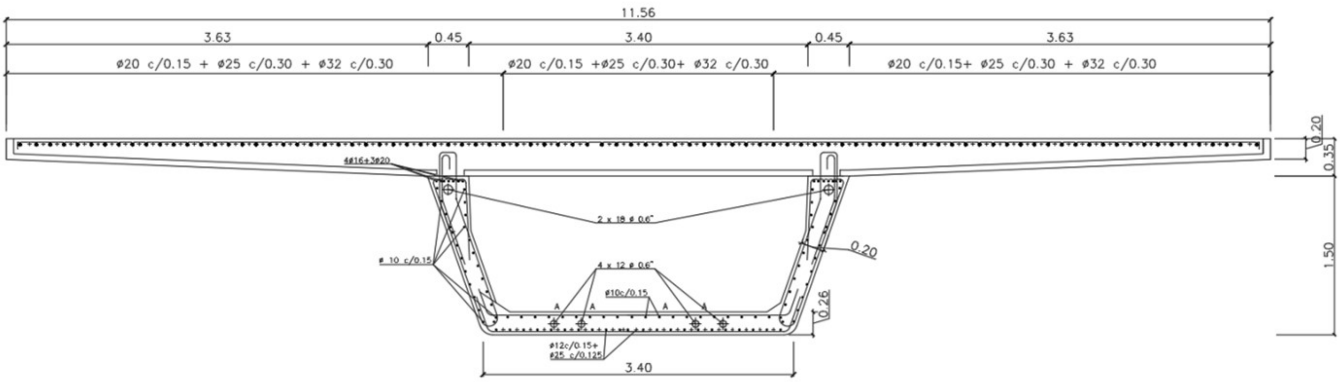


Figure 17. Cross section at pier 1. Section 2.

accurate results in terms of depth fiber and stresses in the slab and in the beam when they are compared with the FAGUS calculation. Differences between the FAGUS software and the neutralization method are less than 1% and differences between FAGUS software and direct calculation are less than 2.5%.

The neutralization method presents advantages in complex cases compared to other methods such as the direct method or using Fagus software. Direct method and Fagus software either do not consider delayed phenomena or it is difficult to introduce or consider them. Neutralization method has lower possibility of error and presents clear advantages for introducing sections of complex geometry such as composite sections consisting of a prestressed concrete beam and a cast-in-place concrete slab with two concretes cast at different times in statically indeterminate concrete bridges with connected precast beams.

5. WORKED EXAMPLES

5.1. Case A

The methodology proposed in the paper is applied to the midspan section of one of the multiple spans of the Viaduct over the Abion River studied in [3]. The structure is a 250 m long continuous bridge with six spans of 25,0 + 40,0 + 3 x 50,0 + 35,0 m. The deck cross-section is a spliced U-shape precast post-tensioned girder with variable depth and a cast-in-place concrete top slab. The analyzed cross-section (referred to as section 1 hereafter) consists of a precast U beam with a depth of 1.41 m and a cast-in-place slab of 13.80 m width and 0.41 m maximum thickness. The characteristic compressive strengths of concrete at 28 days are 50 MPa and 35 MPa, for the beam and the slab, respectively. The layers of prestressing (Y1860S7 according to [3]) and non-prestressing steel (B500S according to [3]) are shown in Figure 16. The area, centroid position and inertia moment about the centroid's horizontal axis of the transformed composite section are $A = 6.160$ m², $y_g = 1.305$ m and I = 2.175m⁴, respectively.

The permanent loads at infinite time considered are the self-weight of the precast beam and the top slab, the prestressing force at the precast beam at infinite time including prestressing losses, and rheological effects. Also, the statically indeterminate bending moment due to permanent loads and prestressing and superimposed loads are considered. The resulting permanent stresses over time have been calculated using the stress redistribution method described in [3] [4], and are shown in table 2. If the live load fraction corresponding to the frequent load combination (ΔM_{Freq} = 11866 m·kN according to [3]) is applied over the permanent stress state, a tensile stress of 2.00 MPa is obtained at the bottom of the precast beam, which is still smaller than the considered tensile strength of concrete (ΔM_{cr} = 15134 m·kN,[3]). Nevertheless, under characteristic load ($\Delta M_{characteristic}$ = 19367 m·kN, [3]) the section would be cracked and not able to support tensile stresses under subsequent frequent load combination. Thus, a consistent analysis requires consideration of cracking for the SLS assessment.

TABLE 2.
Stresses under permanent loads at end time

Location	Section 1 Mid span	Section 2 At pier	Section 3 Mid span
σ_{tsct} (MPa)	-2.79	3.57	-4.93
σ_{bsct} (MPa)	-2.06	2.60	-3.18
σ_{tbct} (MPa)	-7.16	-0.38	-11.76
σ_{bbct} (MPa)	-5.01	-6.61	0.00

To account for cracking, the neutralization method is applied. In Stage I, a neutralization force of PN = 43088.4 kN with an eccentricity of e´ = 1.251 m is necessary to cancel the concrete permanent stresses of the precast beam. Solving equations (3) and (4), with the value of ΔM_{Freq} = 11 866 m·kN yields a neutral axis depth of x = 1.175 m, with $B(x)$ = 4.56 m³, σ_c /x = PN/$B(x)$ = 9448 kN/m³ and σ_{s1} = 23.59 MPa.

The crack width under frequent load combination, relevant for SLS cracking verification, can be calculated with the equations given in Appendix A [EN 1992], which provides (εsm- εcm) = 0.000071, srmax= 313 mm and wk = 0.022 mm (less than 0.2 mm, maximum value according to the corresponding exposure class [1]).

Furthermore, the section is fully compressed under quasi-permanent load combination; so it does not crack and the concrete at the level of the presstressing steel of the beam is in compression.

5.2. *Case C*

In this case, the studied cross-section is the one located over pier 1 of the Viaduct 2+130 of the Project of Enlargement of Road BI-630 (Spain), refer to [3]. The structure is a continuous 79.0 m long viaduct with three spans of 22.0 + 35.0 + 22.0 m. The deck type is a spliced U-shape precast post-tensioned girder with constant depth and a cast-in-place concrete top slab. The analyzed cross section, named section 2 hereafter, is a precast U beam with a depth of 1.50 m and a slab of 11.56 m width and 0.35 m maximum thickness. The characteristic compressive strengths of concrete at 28 days are 50 MPa and 30 MPa for the beam and the slab, respectively. All layers of prestressing (Y1860S7) and non-prestressing steels (B500S) are shown in Figure 17. The area, the centroid position and inertia moment about the horizontal axis on the centroid of the cracked transformed composite section (neglecting the concrete of the slab but considering the non prestressing steel of the slab) are A = 2.102 m², y_g = 0.745 m and I = 0.948 m⁴ respectively.

Under frequent load combination, the concrete stress at the top of the beam is 2.30 MPa and the concrete of the slab is cracked under permanent loads. Stresses due to permanent loads at end time in section 2, are shown in table 2 (the beam is in compression). Thus, the neutralization force and the eccentricity which cancel the stresses of the concrete of the beam due to permanent loads are PN = 7392,9 kN and e´ = 1.138 m. In this case, the following checks must be done at SLS according to EN 1992 [1]: assessment of the crack width at the top of the beam under frequent load combination; and verification that the concrete at the level of the top prestressing steel of the beam is in compression under quasi-permanent load combination.

Solving equations (3) and (4) with the value of ΔM_{Freq} = -5485 m·kN, the neutral axis depth is x = 0,975 m (measured from the bottom of the beam), and the relevant parameters are as follows: B = 0,68 m3, σ_c /x = PN/$B(x)$ = 10950 kN/m³ and σ_{s2} = 24,64 MPa. Introducing these values in equations (5)-(7), the crack control leads to (ε_{sm}- ε_{cm}) = 0.000074, s_{rmax}= 238 mm and w_k = 0,018 mm (at the top of the beam), which is less than 0.2 mm (maximum value according to concrete exposure class [1]).

Following the same procedure under quasi-permanent load combination (ΔM_{quasi} = -1063 m·kN) the neutral axis depth is x = 1,37 m from the bottom of the beam, so the concrete at the level of the top prestressing steel of the beam is in compression.

6.
INFLUENCE OF THE DEFINITION OF THE CONCRETE MODULUS OF ELASTICITY

All the above calculations of the transformed section have been carried out using the instantaneous modulus of elasticity of concrete at infinite time, E_{cb} (t_∞). Nevertheless, it is not totally clear which definition of the modulus of elasticity (either instantaneous or age-adjusted) is conceptually more consistent due to the fact that cracking under both quasi-permanent and frequent loads is not an instantaneous event. In principle, the real modulus of elasticity of the concrete for quasi-permanent loads seems close to the age-adjusted modulus, but an intermediate value between the age-adjusted and instantaneous moduli seems more reasonable for frequent actions.

The choice of the most accurate definition of the concrete modulus for each case is not an easy question and a sensitivity analysis has been carried out in this Section to check the influence of such a parameter, used for the cracked section analysis of Stage II. The same value of the modulus of elasticity must be taken for Stages I and II so that superposition can be applied. Five cases have been studied, including the instantaneous modulus of elasticity at 28 days and end time, and the age-adjusted modulus of elasticity at 1, 2 and 3 years. In order to determine the age-adjusted modulus of elasticity between two-time instants $t_1 < t_2$, the following formulation [3] is applied:

$$E_c(t_2, t_1) = \frac{E_c(t_1)}{1+\chi(t_2, t_1)\,\varphi(t_2, t_1)} \quad (7)$$

where $\chi(t_2, t_1)$ and $\varphi(t_2, t_1)$ are the aging and the creep coefficient in the period (t_2, t_1).

The study is applied to the midspan cross-section of the Viaduct 2+130 of the Project of Enlargement of Road BI-630 (Spain) [3]. The analyzed cross-section, named section 3 hereafter, is a precast pre-tensioned U-shaped beam with a depth of 1.50 m and a cast-on-site slab of 11.56 m width and 0.35 m maximum thickness. The geometry of the cross-section is represented in Figure 16. The values of the modulus of elasticity are shown in Table 3.

TABLE 3.
Modulus of elasticity of the concrete

Modulus of Elasticity	Ec Instantaneous t = ∞	Ec Instantaneous t = 28 days	Ec Delayed t = 1 year	Ec Delayed t = 2 year	Ec Delayed t = 3 year
E_{cbeam} (MPa)	43544	38640	17117	16142	15746
E_{cslab} (MPa)	37819	33560	12937	12006	11618

The area, the position of the centroid and the inertia moment about the horizontal axis at the centroid of the transformed composite section are shown in Table 4 depending on the different value of the modulus of elasticity of the concrete. Permanent concrete stresses at the slab and the beam are shown in Table 2. It should be noted that the permanent loads have been increased with respect to the values of the project to obtain zero stress at the bottom of the beam as in the real situation the section is fully compressed.

Table 4 summarizes the neutralization forces (PN, e´) for section 3 depending on the value of the modulus of elasticity. As it is shown, the highest neutralization forces are obtained with the instantaneous modulus of elasticity at end time, while the lowest values are obtained with the age-adjusted modulus at 3 years, but the difference between extreme values is around 7%.

The increase of the bending moment due to quasi-permanent and frequent combination of loads are 2356 and 8637 m·kN respectively. The neutral axis depth (x) under frequent load combination, measured from the top of the section, is shown in Table 4. It should be noted that the smallest values correspond to instantaneous modulus of elasticity and the highest values correspond to age-adjusted modulus of elasticity of concrete, which means that the higher the modulus of elasticity, the lower the position of the neutral axis.

Regarding crack width control of the beam, it can be observed from Table 4 that the values are very similar, being the difference between extreme values 4%. The highest values correspond to the instantaneous modulus of elasticity at end time. Regarding checking under quasi-permanent load combination, the cross-section remains fully compressed when the age-adjusted modulus of elasticity is used, whereas is used, the bottom of the beam is tensioned if the instantaneous modulus of elasticity is considered. Nevertheless, the concrete at the layer of the prestressing steel of the beam is compressed, as shown in Table 4.

TABLE 4.
Analysis of the section with different values of modulus of elasticity of the concrete

	Ec Instantaneous t = ∞	Ec Instantaneous t = 28 days	Ec Delayed t = 1 year	Ec Delayed t = 2 year	Ec Delayed t = 3 year
A(m²)	4.791	4.815	4.685	4.671	4.663
I (m^4)	1.924	1.936	1.995	2.001	2.003
yg(m)	1.291	1.290	1.248	1.244	1.241
PN (kN)	48471	48687	45833	45534	45370
e´ (m)	1.472	1.472	1.460	1.458	1.457
X(m) Freq Comb	0.835	0.875	1.125	1.150	1.160
w_k (mm)	0.0865	0.0856	0.0852	0.0827	0.0818
X(m) Quasi Comb	1.750	1.775	1.800	1.800	1.800
σ_p (MPa) Quasi Comb	-1.00	-2.04	-6.29	-6.66	-6.82

As a conclusion, the use of the instantaneous modulus of elasticity is on the safe side as it leads to higher values of crack width of the beam and higher neutral axis depth under both quasi-permanent and frequent load combinations. Despite the higher compressive stresses, they remain far below the established stress check limits. However, it is proved that the results have a very limited dependence of the value of the modulus of elasticity of the concrete.

7. CONCLUSIONS

This paper has presented an analytical method for serviceability and crack control in composite sections consisting of a prestressed concrete beam and a cast-on-site concrete slab, both cast at different times. The approach is based on the neutralization method for prestressed concrete sections [7] 20] and has been extended and adapted to composite sections with more than one concrete. The neutralization method provides good agreement regarding the depth of neutral fiber and the stresses in the slab and beam.

The proposed method's key advantage lies in its applicability to composite sections, accounting for prior stress redistribution from concrete shrinkage, creep, and prestressing steel relaxation while also providing a quick tool for assessing the serviceability analysis and offering an alternative to more complex approaches.

The method has been exposed for three cases of practical relevance: midspan sections subjected to positive bending with cracking of the beam, support sections subjected to negative bending in which the top slab is uncracked under permanent loads but undergoes cracking with the application of live loads, and support sections subjected to negative bending in which the top slab is already cracked under permanent loads.

When dealing with cracked positive moment areas, the 'in situ concrete layer' assumes the role of prestressing steel during the neutralization process, requiring consideration of its dimensions using the transformed section method. This approach results in zero stresses only within the precast beam, simulating a homogeneous section. Similarly, support sections experiencing negative bending moments with an uncracked slab under permanent loads follow a comparable methodology, considering the concrete of the beam as "the in situ concrete layer" in positive moment areas, because the concrete of the beam is compressed in all stages.

Conversely, support sections encountering negative bending moments with a cracked slab under permanent loads are approached differently. In this scenario, considering the concrete of the slab as ineffective leads to treating the section as prestressed non-composite, with the reinforcement steel of the slab positioned externally to the concrete. This strategy allows for the treatment of the section as a homogeneous one, distinct from the former cases due to the inefficacy of the cracked slab's concrete.

The influence of the choice of the modulus of elasticity of the concrete has been analyzed, showing that its influence on the results is rather limited. Moreover, the use of the instantaneous modulus of elasticity gives results of neutral axis depth and crack width on the safe side compared to the values obtained with the age-adjusted modulus of elasticity. Thus, it can be concluded that it is not necessary to deal with the inherent difficulties to determine the age-adjusted modulus of elasticity of the concrete for permanent or frequent situations.

Acknowledgements

To Miguel Estaún Ibañez, Civil Engineer, and Director of StrucREs, an engineering company specialized in continuous precast bridges, for providing all the documentation related to two examples of continuous precast projects: Project of the Viaduct over the Abion River and the Viaduct located in chainage 2+130 of the Project of Enlargement of Road BI-630 (Spain).

Appendix A. Crack width calculation according to EN 1992

In order to calculate crack width, the following formulation of the Eurocode 2 EN 1992 [1] is applied:

$$\omega_k = s_{rmax}\,(\varepsilon_{sm} - \varepsilon_{cm}) \tag{8}$$

$$s_{rmax} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_3 \cdot k_4\,\frac{\Phi}{\rho_{p,eff}} \tag{9}$$

$$(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\sigma_c - k_t\,\dfrac{f_{cteff}}{\rho_{p,eff}}\,(1 + \alpha_e \cdot \rho_{p,eff})}{E_S} \geq 0.6 \cdot \frac{\sigma_s}{E_s} \tag{10}$$

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