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Regarding the upper limit of the shear stiffness in battened members in Eurocode 3

En relación al límite superior de la rigidez a cortante propuesta en el Eurocódigo 3 para piezas compuestas empresilladas

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ABSTRACT

Part 1-1 of Eurocode 3 provisions are applicable to built-up members of buildings formed by identical chords with lacings in two planes. This paper proposes a correction on the formulation provided by Eurocode 3 in regard to the upper limit of the shear stiffness in battened members. This upper limit is commonly used as a first input in the pre-design procedure of built-up members. The upper limit proposed by Eurocode 3 corresponds to an approximation included in the German regulation DIN 4114-1 in which the design of battened members was done using a fictitious slenderness, a concept that is not used in the current Part 1-1 of Eurocode 3. As a consequence of this, Eurocode 3 decreases the theoretical upper limit of the shear stiffness of these members leading to an artificial increment of the design moment. Furthermore, for educational reasons a correction is proposed in order to use a coherent theoretical support for the design of steel structures. A detailed justification is presented.

KEYWORDS: Shear stiffness; upper limit; battened members; Eurocode 3.

RESUMEN

La parte 1-1 del Eurocódigo 3 es aplicable a piezas compuestas formadas por cordones idénticos unidos con dos planos de enlace. Este artículo propone una corrección en la formulación propuesta por el Eurocódigo 3 para el límite superior de la rigidez a cortante en piezas compuestas empresilladas. El límite superior se suele usar en el predimensionamiento de los elementos empresillados. El límite superior de la rigidez a cortante propuesto por el Eurocódigo 3 corresponde a una aproximación incluida en la norma DIN 4114-1 en la que el dimensionamiento de las piezas compuestas se hace considerando una esbeltez complementaria, un concepto en el que no se basa la Parte 1-1 del actual Eurocódigo 3. Como consecuencia, el Eurocódigo 3 disminuye el límite superior teórico de la rigidez a cortante de los elementos empresillados, lo que conlleva un incremento artificial del momento de diseño. Además, desde un punto de vista académico, es interesante que el dimensionamiento de este tipo de elementos estructurales se base en un soporte teórico coherente. Se presenta aquí una justificación detallada.

PALABRAS CLAVE: Rigidez a cortante; límite superior; elementos empresillados; Eurocódigo 3.

1. INTRODUCTION

There are two types of built-up columns, laced and battened members (figure 1). Battened members are formed by identical parallel chords linked by planes of battens (usually n=2) placed at certain locations, uniformly spaced along the element. The chords are usually formed by steel angles, channels or I-sections.

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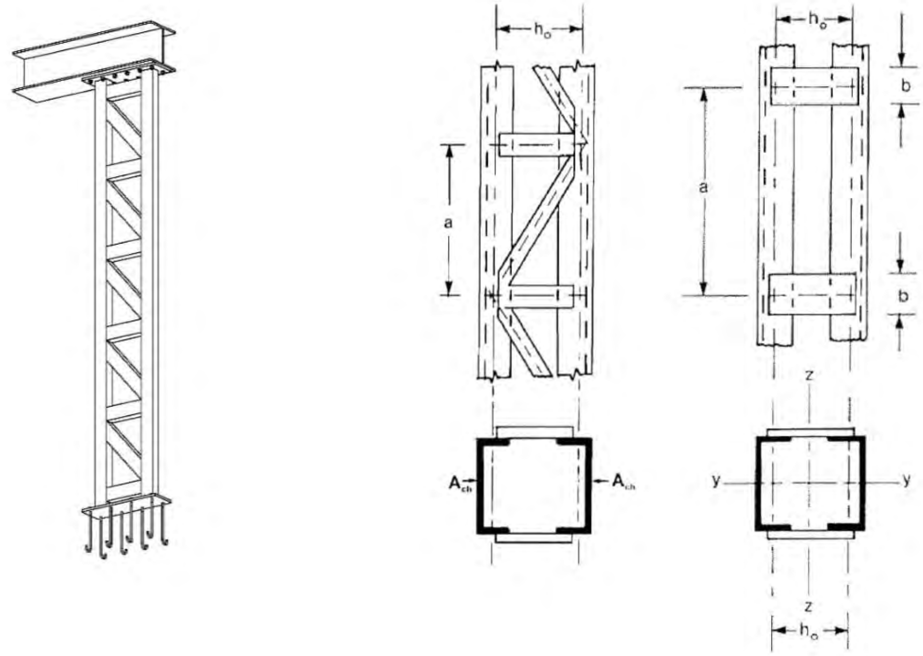

Built-up column | Laced column | Battened column

Figure 1. Built-up column members (Adapted from Gil-Martín y Hdz.-Montes 2020).

These members, which can be used to strengthen reinforced concrete columns [2] [3], are widely used in steel construction as bracings, columns of buildings, transmission towers, etc. The collapse of this type of elements may occurs as a results of overall buckling, local bucking of each chord between two consecutive battens or due to the lack of strength of the battens.

The particularity of these members as regards the axial buckling strength is that, due to their reduced shear rigidity, the shear deformations cannot be neglected which lead to a reduction of the critical axial buckling load [4].

This work is focused on building´s battened built-up members subjected to constant axial loading, to which formulation in Part 1-1 of Eurocode 3 [5] is applicable.

2. PROPORTIONING OF BATTENED BUILT-UP MEMBERS ACCORDING TO PART 1-1 OF EUROCODE 3.

The Eurocode 3 (EC3) [5] formulation is applicable to built-up members formed by identical parallel chords linked by two planes of battens (*n*= 2). The member has to consist of at least 3 modules having the same length, *a*.

For the calculation of hinged ends built-up members buckling transversely to the non-material principal axis, a bow imperfection of L/500 is adopted and the shear deformation of the member in the plane of the battens is considered through the shear stiffness, S_v.

To proportion these members it is necessary to obtain the value of the maximum moment at the middle length of the member considering second order effects, given by ([5], §6.4.1):

$$M_{Ed} = N_{Ed}\frac{L}{500}\frac{1}{1-\frac{N_{Ed}}{N_{cr}}-\frac{N_{Ed}}{S_v}} \qquad (1)$$

being:

- N_{Ed} the design value of the compression force in the built-up member
- N_{cr} the effective critical force of the built-up member
- S_v the shear stiffness of the panel

In the case of battened compression members, the value of S_v can be obtained equalling the in-plane deflection due to shear at a point of the chord equidistant between two consecutive battens to the deflection of an ideal plate of steel *0.5a* width. It is assumed that the stress distribution in the ideal plate is uniform, being *Ω* its area.

Both models are represented in figure 2. Figure 2a corresponds to a segment of the battened member subjected to shear V. The deformed shape is also drawn in by a dashed line. In figure 2b an ideal steel plate subjected to shear and its corresponding deformation are represented.

In model 2a the deflection at point 1 is obtained as sum of the deformation of the chord along its length (cantilever *0.5a* length with punctual load of *V/2* at the free end) plus the deformation induced by the deformability of the battens (obtained as the rotation at the end of the battens multiplied by *0.5a*). See Eq.(2).

$$\delta = \underbrace{\frac{Va^2h_0}{24E\left(nI_b\right)}}_{\text{battens}} + \underbrace{\frac{Va^3}{48EI_{ch}}}_{\text{chord}} \qquad (2)$$

In the expressions above, h_0 is the distance between the centroids of chords, *a* is the distance between centrelines of consecutive battens, n is the number of planes of battens (n=2), I_{ch} is the second moment of area of one chord, I_b is the second moment of area of one batten, E is the Modulus of Elasticity and G is the shear modulus of the steel.

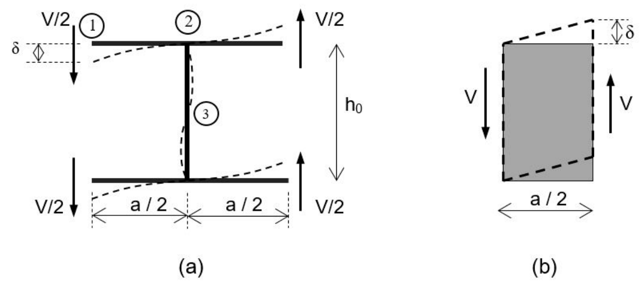

Figure 2. Models to determine the shear stiffness. a) Segment of battened member. b) Ideal steel plate with equivalent deformation.

From the model shown in figure 2b:

$$\tau = \frac{V}{\Omega} = G\gamma \rightarrow \gamma = \frac{V}{G\Omega} = \frac{V}{S_v} \rightarrow \delta = \gamma\frac{a}{2} = \frac{Va}{2S_v} \quad (3)$$

Equalling Eq. (2) and Eq. (3), the following expression for the shear stiffness is obtained:

$$S_v = \frac{1}{\frac{a}{12E}\left(\frac{h_0}{nI_b} + \frac{a}{2I_{ch}}\right)} = \frac{24EI_{ch}}{a^2\left(\frac{2h_0I_{ch}}{nI_ba} + 1\right)} \quad (4)$$

The expression given by Eq. (4) accounts for both, the stiffness of chords and battens.

However, for the proportioning of new built-up battened members, in which the battens have not yet been defined, it is useful to neglect the flexibility of the battens to obtain the upper limit of S_v.

If the first term in Eq. (2) is neglected (i.e. battens are supposed infinitely rigid), the deflection δ in the model of figure 2a is the smallest, and so, the corresponding value of S_v in model of figure 2b) -i.e. Eq. (3)- is the highest. Therefore, the upper value of the shear stiffness (S_v) is given by:

$$\delta = \frac{Va^3}{48EI_{ch}} = \frac{Va}{2S_v} \rightarrow S_v = \frac{24EI_{ch}}{a^2} \quad (5)$$

Unifying Eq. (4) and Eq. (5), the following expression is obtained:

$$S_v = \frac{24EI_{ch}}{a^2\left(\frac{2h_0I_{ch}}{nI_ba} + 1\right)} \leq \frac{24EI_{ch}}{a^2} \quad (6)$$

In the above expression of S_v both sides of the inequation are formulated based on the same mechanical model.

However, EC3 [5] in Section 6.4.3 gives the following expression for the shear stiffness of battened members:

$$S_v = \frac{24EI_{ch}}{a^2\left(\frac{2h_0I_{ch}}{nI_ba} + 1\right)} \leq \frac{2\pi^2EI_{ch}}{a^2} \quad (7)$$

As can be seen, the value of S_v is the same in Eq. (6) and Eq. (7) but they differ in the upper limit. Comparison between Eq. (6) and Eq. (7) shows that formulation in EC3 [5] considers the 82% of the actual upper limit of S_v. This reduction of the shear stiffness, which is applied in practice [6], leads to an artificial increment of the design moment -Eq. (1)-.

Eq. (7) has also been adopted in national regulations based on the EC3 [5], such as the Spanish Standard EAE [7].

3.

PROPORTIONING OF BATTENED BUILT-UP MEMBERS BASED ON THE CONCEPT OF EQUIVALENT SLENDERNESS RATIO (DIN 4114 STANDARD).

To understand the value of the upper limit proposed by EC3 [5] -i.e. Eq. (7)- it is necessary to go back to the regulation DIN 4114 [8], in which the elastic critical buckling load of a pin ended column about the non-material principal axis (i.e. considering shear deformation) is obtained using the Euler´s formulation but considering an equivalent slenderness ratio, $\hat{\lambda}$, $\sqrt{\hat{\lambda} = \lambda^2 + \lambda_1^2}$, where λ is the slenderness ratio of the battened member about the non-material principal axis (defined as the ratio between the effective length of the member and the radius of gyration about the axis under consideration) and λ_1 is an additional slenderness.

The concept of equivalent slenderness ratio is also adopted by regulations such as the Spanish building construction code (CTE) [9] and the European standard of overhead electrical lines [10].

Assuming that the built-up column is formed by two identical parallel chords, the equivalent slenderness ratio, $\hat{\lambda}$, is obtained as:

$$N_{cr,shear} = \frac{N_{cr}}{1 + \frac{N_{cr}}{S_v}} = \frac{\pi^2E2A_{ch}}{\hat{\lambda}^2} \Rightarrow \hat{\lambda}^2 = \lambda^2 + \frac{\pi^2E2A_{ch}}{S_v} \quad (8)$$

$$\text{with} \quad N_{cr} = \frac{\pi^2E2A_{ch}}{\lambda^2}$$

where N_{cr} is the critical buckling load and $N_{cr,Shear}$ is the critical buckling load accounting for the effect of the shear force (see §2.17 of [4]). A_{ch} is the cross section area of one chord.

As can be seen in Eq. (8) the slenderness ratio λ of built-up members is increased with an additional term, the slenderness λ_1, in order to account for the shear force acting in the member. The value of this additional slenderness is:

$$\hat{\lambda}^2 = \lambda^2 + \frac{\pi^2E2A_{ch}}{S_v} = \lambda^2 + \lambda_1^2 \Rightarrow \lambda_1^2 = \frac{\pi^2E2A_{ch}}{S_v} \quad (9)$$

Taking into account the value of S_v given by Eq. (4) and neglecting the flexibility of the battens (Eq. (5), the following expression is obtained for λ_1:

$$\lambda_1^2 = \frac{\pi^2 E 2A_{ch}}{S_v} > \frac{\pi^2 E 2A_{ch}}{\frac{24EI_{ch}}{a^2}} = \frac{\pi^2}{12}\frac{a^2}{i_{ch}^2} \quad \text{with } i_{ch}^2 = \frac{I_{ch}}{A_{ch}} \qquad (10)$$

For the sake of simplicity, instead of the value given in Eq. (10) the DIN 4114 code [8] adopted as value for the slenderness λ_1 the following expression:

$$\lambda_1^2 = \frac{a^2}{i_{ch}^2} \qquad (11)$$

which implies the following approximation (see Eq. (10) and Eq. (11)):

$$\frac{\pi^2}{12} \sim 1 \rightarrow 2\pi^2 \sim 24 \qquad (12)$$

So, if approximation in Eq. (11) is introduced in Ec. (10) the upper limit proposed by EC3 [5] -i.e. Eq. (7)- for S_v can be deduced.

4. PRACTICAL CASE

Let's study a battened column with two identical chords. The column is pinned connected at both ends, having a length of 3 m (= 3000 m, buckling length). The axil factored load, N_{Ed}, is 1200 kN. Each chord is an IPN200 (A_{ch}=2850 mm^2 -cross-sectional area of one chord-, I_{ch}=1.42·106 mm^4 -second moment of area of one chord respect to its weak axis-) and the distance between centroids of chords is 150 mm (h_0). Five levels of battens are considered, i.e. a=3000/4=750 mm. Structural steel S235 is assumed (figure 3).

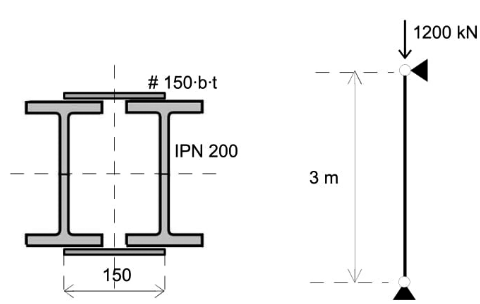

Figure 3. Practical case of battened column.

The shear stiffness of the battened column, S_v, as a function of the second moment of area of one batten, I_b (defined as $tb^3/12$, see Fig. 3), has been represented in figure 4. As can be seen in this figure, the upper limit proposed by both EC3 [5] and EAE [7] for S_v leads to a slight reduction of the shear stiffness of the battened column in the range where the flexibility of the battens can be neglected.

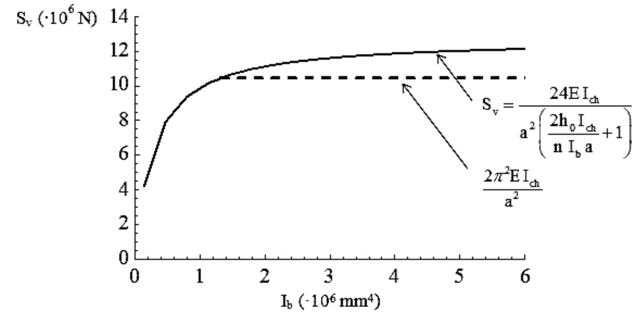

Figure 4. S_v as a function of I_b for battened column in figure 3.

Once the maximum moment at the mid-length of the built-up member including second-order effects, M_{Ed} given by Eq. (1), is known both the design force $N_{ch,Ed}$ of the chord at mid-length and the internal shear force, V_s, at the end of the built-up member (where the maximum shear forces occur) can be obtained as:

$$N_{ch,Ed} = \frac{N_{Ed}}{2} + \frac{M_{Ed}h_0}{2I_{eff}}A_{ch} \quad \text{with} \quad I_{eff} = \frac{A_{ch}h_0^2}{2} + 2I_{ch}$$
$$V_S = \frac{\pi M_{Ed}}{\ell} \qquad (13)$$

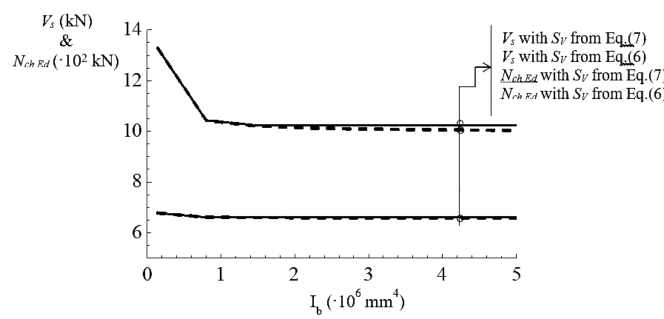

Figure 5. $N_{ch,Ed}$ and V_s as a function of Ib for battened column in figure 3.

Figure 5 shows the values of $N_{ch,Ed}$ and V_s as a function of I_b. As can be seen, from a practical point of view, there is very little difference in the values of the design forces of the battened member whatever the upper limit of S_v (Eq. (6) or Eq. (7)) is adopted. Even more, the limit corresponding to Eq. (7) leads to slightly higher values of the design forces, which is on the side of safety.

5. CONCLUSION

In Standards based on the concept of equivalent slenderness ratio for the proportioning of battened built-up members, such as the DIN 4114 [8], the approximation of 0.82 by 1.0 (i.e. $\pi^2/12$ by 1) makes sense since it simplifies the formulation. Nevertheless, because the proportioning of these members in the current European standard of steel structures EC3 [5] is not based on the philosophy of the DIN 4114 [8] and does not use the concept of equivalent slenderness, the above simplification is not needed. Even when, from a practical point of view, there is no difference between the upper limit adopted for the shear stiffness of battened built-up members, it seems more reasonable, from a conceptual point of view, the adoption of Eq. (6) instead of Eq. (7) for the shear stiffness of battened built-up members.

References

[1] L.M. Gil-Martín y E. Hernández-Montes, Estructuras Metálicas y Mixtas, Colección Seinor. Colegio de Ingenieros de Caminos, Canales y Puertos. Editorial Garceta (en imprenta), 2020.

[2] D. Fernández Montes, E. Díaz Heredia, (2017), Aspectos particulares que condicionan la efectividad de los refuerzos de pilares, Hormigón y Acero, 68(282), 97-106. https://doi.org/10.1016/j.hya.2017.04.013.

[3] E. Calderón Bello, E. Díaz-Pavón Cuaresma, (2018), Dificultades en el planteamiento del refuerzo de pilares de hormigón armado, Hormigón y Acero, 69(284), 49-58. https://doi.org/10.1016/j.hya.2017.09.001.

[4] S. P. Timoshenko and J. M. Gere, Theory of elastic stability, McGraw-Hill Book Co., Inc., New York, (1963).

[5] EN 1993-1-1: Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings, European Committee for Standardization, 2005.

[6] Orbán F., Farkas J. (2013) Optimum Design of Steel Built-Up Compression Members. In: Jármai K., Farkas J. (eds) Design, Fabrication and Economy of Metal Structures. Springer, Berlin, Heidelberg.

[7] EAE: Instrucción de Acero estructural. Ministerio de Fomento, 2011.

[8] DIN 4114-1,Steel Structures. Stability (buckling, overturning, bulging). Method of calculation, regulations. German Institute for Standardisation (Deutsches Institut für Normung), 1952.

[9] CTE: Código Técnico de la Edificación. Ministerio de Vivienda, 2017.

[10] UNE EN 50341-1: Overhead electrical lines exceeding ac 1 kv - Part 1: general requirements - common specifications, 2017.